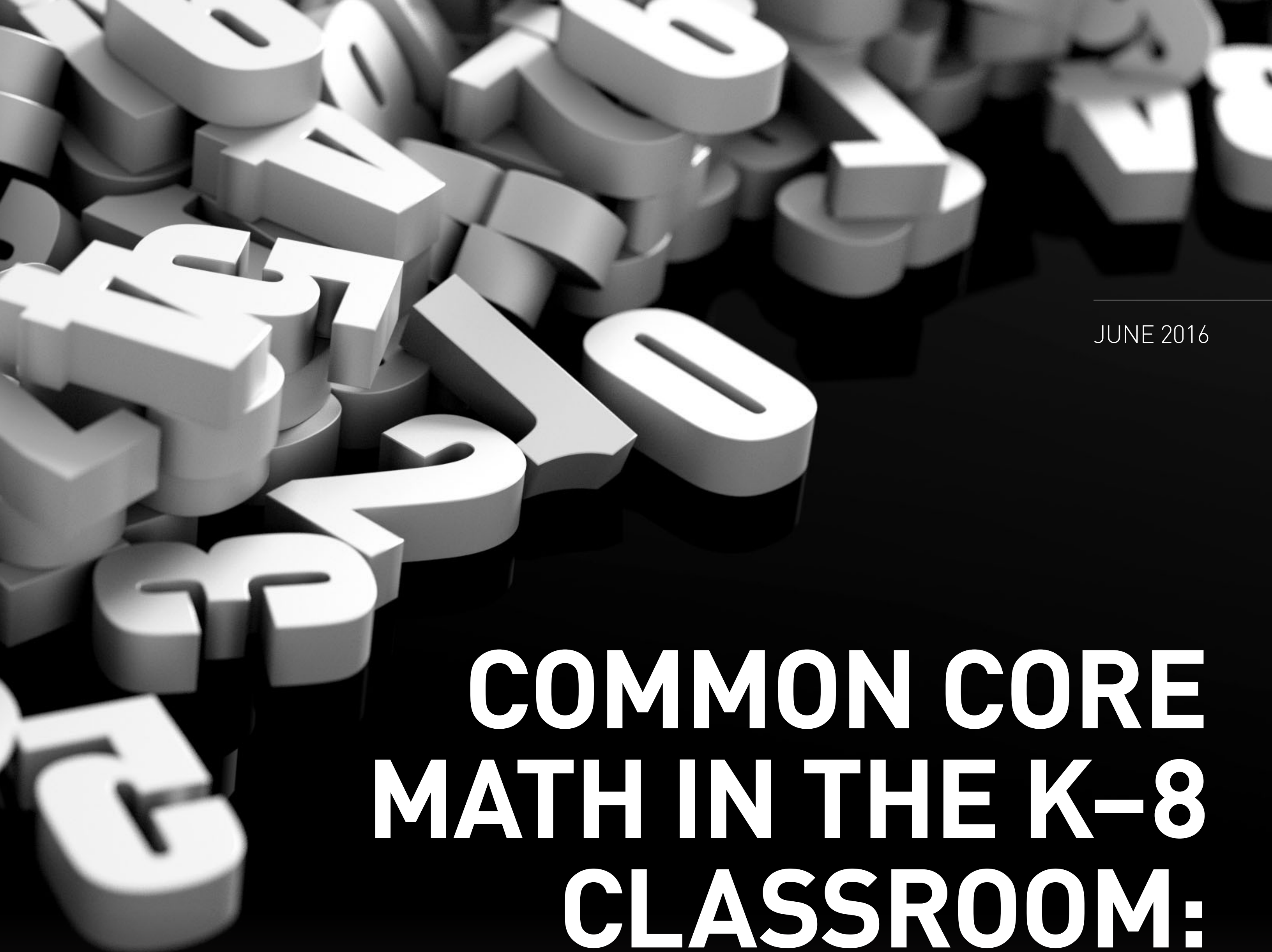

JUNE 2016

COMMON CORE MATH IN THE K–8 CLASSROOM:

RESULTS FROM A NATIONAL TEACHER SURVEY

JENNIFER BAY-WILLIAMS
WITH ANN DUFFETT AND DAVID GRIFFITH

THOMAS B. FORDHAM INSTITUTE
ADVANCING EDUCATIONAL EXCELLENCE

FOREWORD AND EXECUTIVE SUMMARY BY
AMBER M. NORTHERN AND MICHAEL J. PETRILLI

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Foreword

Amber M. Northern and Michael J. Petrilli

In 2010, when the final Common Core State Standards (CCSS) were unveiled, our content experts found them worthy of praise, awarding the math standards an A-minus and the English language arts standards a B-plus. That meant CCSS was "clearly superior" to the standards in the vast majority of states—and that the vast majority of American children would be better off if their schools taught them the content and skills they set forth.

Since then, we've remained steadfast in our belief that the standards, if adequately implemented and supported, could improve the educational trajectories and life prospects of all students.

We've been busy the last six years studying how this implementation has been going. Between our national and Ohio offices, we've published an average of two studies per year on Common-Core-related topics, totaling an even dozen. Yet our earnest, unequivocal support of the CCSS means that we're subject to skepticism: Is Fordham wearing rose-colored glasses when it comes to how it's going?

If you've read any of our dozen reports, you already know that the answer is no. We haven't been shy about exposing implementation warts. In 2010, for instance, we raised a question in a white paper that *none* of our Common Core friends wanted to talk about at the time (and still don't): How should the Common Core be governed going forward (including whether and how these standards may one day be updated)? In 2012, we reported that CCSS implementation was not going to be done on the cheap. In 2013, we explained how English language arts teachers in Common Core states still viewed instruction with a skills-based approach, versus one focused on content; that same year, we released another study reporting that a measly 12 percent of superintendents in our home state of Ohio described themselves as "high-level implementers." And in 2014, we laid out the many struggles that "early implementer" districts faced in executing the CCSS.

The present study also seeks to offer relevant, honest and—we hope—practical research on CCSS implementation. Here, we're interested in whether teachers responsible for elementary and middle school math instruction in Common Core states have changed what and how they teach—and whether they're seeing improvements in students' math understanding as a result. The teacher survey that forms the basis of this report was developed, in part, to help guide implementation efforts on the ground. (This is the math parallel to our English language arts study released in 2013.) This study joins a growing body of research showing that teacher familiarity with the Common Core standards is growing, as is their acceptance of them.[1, 2, 3] But there's plenty new here as well.

Successfully undertaking survey research that "speaks" to K–8 math teachers requires analysts who know the subject, who know teachers, and who also know survey design and analysis. Our trio of authors fits that bill. Lead analyst Jennifer Bay-Williams is professor and department chair of middle and secondary education at the University of Louisville. She's the past president of the Association of Mathematics Teacher Educators and has published many articles in refereed journals (and a widely used book series for K–8 mathematics teachers) on how this subject is taught and learned. Just as important, she walks the walk by assisting teachers throughout Kentucky and other states who are in the throes of CCSS-M implementation.[4]

Bay-Williams is joined by Ann Duffett, a twenty-year veteran in the field of public opinion research. She is the former senior vice president at Public Agenda, venturing out in 2005 to found the FDR Group with her partner, Steve Farkas. Ann and Steve were Fordham's first-ever contractors in the late 90s; we respected their work then and continue to in 2016.

Jennifer and Ann are joined by Fordham Research and Policy Associate David Griffith. David holds a master's degree in public policy from Georgetown University and is a former high school social studies teacher. He excels in asking the right questions, pushing back against conventional wisdom, and writing user-friendly prose.

This able trio has produced a fine report that you'll want to read in its entirety. Because you'll find a thorough executive summary following these remarks, we won't rehash all of its findings. But here are five key takeaways:

1. **Most teachers are partial to the Common Core, but they don't think all of their students and parents are.** Most teachers view the standards positively, believing that they will enhance their students' math skills, prepare them to succeed in college, and bolster their ability to compete in a global economy. Further, most believe that the CCSS-M are an impetus for improving their *own* content knowledge. At the same time, teachers' thoughts on the views of students and parents are considerably less rosy. They say that pupils are "frustrated" by having to learn multiple methods of solving a problem and worry that some have math anxiety, especially in grades 6–8. A whopping 85 percent say that "reinforcement of math learning at home is declining because parents don't understand the way that math is being taught." We know that teachers are the primary vehicle through which parents learn about the CCSS, so this raises a key question: How can they better help parents support their children's success in math?

2. **Kids seem to be hitting a wall in middle school**. Or, possibly, their teachers are. Overall, middle school teachers tend to have a more negative assessment of their students' math abilities and the broader impact of the standards. We can't know why. Perhaps it's the obvious reason: Middle school standards are simply tougher than elementary standards. Perhaps it's because more teachers in middle school have math degrees (versus elementary education degrees) and thus better grasp the math prowess needed in the upper grades. Or maybe it's because today's middle school students did not "grow up" with these standards in earlier grades, and the transition has been difficult. The logical question is this: If the latter is true, will this transition problem eventually work itself out?

   We were surprised to find that elementary teachers tend to have more positive views of the potential benefits of CCSS-M and their impact on students. After all, we've heard many anecdotal reports of elementary teachers who feel that these standards are "developmentally inappropriate," and it's no secret that many primary teachers haven't themselves studied a great deal of math. Yet 61 percent of K–2 teachers say they have *fewer* or about the same number of "students who have math anxiety" than before the CCSS-M, and 68 percent agree that "students are developing a stronger capacity to persevere in math and come up with solutions on their own." It's the middle school teachers who report more distress.

3. **Teaching multiple methods can yield multiple woes**. The CCSS-M's Standards for Mathematical Practice require that students "check their answers to problems using a different method." And sure enough, 65 percent of K–5 teachers are teaching multiple methods more now than before the CCSS-M were implemented. Our focus group data and open-ended survey responses also reveal this to be one of the biggest lessons they've drawn from Common Core: Teach multiple ways to solve a problem. Makes sense to us. But this admonition has had

ramifications for both students and their parents. One likely reason that math learning is suffering at home is that parents simply don't know "multiple methods" themselves—they know the method they were taught two or three decades ago. Our authors recommend one solution (have students practice their *preferred* method at home), but surely there are other ways to teach students conceptual understanding without flummoxing them or their moms and dads.

4. **The math wars aren't over.** The Common Core math standards seek to bring a peaceful end to the "math wars" of recent years by requiring equal attention to conceptual understanding, procedural fluency, and application (applying math to real-world problems). Yet striking that balance has not been easy. We see in these results several examples of teachers over- or under-emphasizing one component to the detriment of another. A recent RAND study found the same thing and recommended that math teachers be given better guidance on how to balance the three areas in classrooms.[5]

5. Finally, the good news: **Teachers are teaching Common Core math content at the grade levels that the CCSS-M specify**. Though that may seem anticlimactic, it is noteworthy that teachers are able to identify from a list of topics (some of which are decoys) those that reflect the standards—and that they report teaching those topics at the grade levels where they're supposed to be taught. This finding resonates with a study earlier this year by Thomas Kane and colleagues in which 85 percent of teachers reported having good or excellent knowledge of the standards for the grades and subjects that they teach.

   Once upon a time, teachers shut their doors and did their own thing. Now we have many instructors teaching to the same high standards nationwide. This is something to celebrate.

These results should inspire several mid-course adjustments. The report offers various suggestions on this front, such as the need to clarify a role for memorization in the early grades. (See "Implications" for more.) We hope that local and state officials, teachers, and teachers of teachers seize that opportunity.

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There is a growing body of research on CCSS implementation. We're learning what changes are occurring in classrooms, what misconceptions and struggles teachers are having, and what district and school leaders might do to address them.

Still, many people understandably find the shift to CCSS challenging, frustrating, and exasperating. (Politics has also reared its unlovable but inevitable head.) Implementation of a big change in practice is never easy in our large, decentralized, and change-resistant education system.

We've been around long enough to see a number of major reforms—all of which seemed controversial and hard to implement at the outset—achieve considerable good over the long run, including some that were prematurely deemed ineffective. (The "small schools" movement comes to mind.) What other policies, structures, or interventions might have borne fruit had we waited long enough to see them out?

The data here are mostly encouraging, even as they give us plenty of ideas for how to do better. If most teachers are implementing the standards, as they claim, we should expect to see improvements in student achievement going forward. Teachers are weary of the pendulum swings in schools; we must show more patience with the Common Core than we've shown in the past. Let's actually see this thing through.

Acknowledgments

This research was made possible through the generous support of the Brookhill Institute of Mathematics, the Exxon Mobil Corporation, the Overdeck Family Foundation, the Bill & Melinda Gates Foundation, and our sister organization, the Thomas B. Fordham Foundation.

Many thanks to co-authors Jennifer Bay-Williams, Ann Duffett, and David Griffith. These were difficult data to analyze and interpret, and we are grateful for their efforts in producing such a user-friendly report. Thanks also to Gabrielle Martino, who helped to design the survey and conduct preliminary analysis, and to Jason Zimba, who reviewed the report's draft. Any errors of analysis or fact are the responsibility of the authors alone.

At Fordham, we extend appreciation to National Research Director Dara Zeehandelaar, for help with project development and survey design; President Emeritus Chester E. Finn, Jr., for providing feedback on report drafts; Alyssa Schwenk, for managing dissemination; and Jonathan Lutton, for handling production and social media. We also express thanks to Shannon Last, who served as copy editor, and Edward Alton, who designed the report's layout.

We are especially grateful to the math teachers across the country who took the time to complete our survey or participate in a focus group. Without them, this report would not have been possible.

Executive Summary

Amber M. Northern and Michael J. Petrilli

Successful implementation of the Common Core State Standards for Mathematics (CCSS-M) should result in noticeable changes in primary and middle school classrooms across the United States. After all, compared to most of the state standards that they replaced, the CCSS-M focus on fewer topics, link concepts across grades more effectively, and increase attention to rigor. In turn, these "shifts" are intended to support more focused, coherent, and rigorous instruction, ostensibly leading to greater student learning. But are the intended and expected changes in educational practice actually occurring at the classroom level?

To find out, the Thomas B. Fordham Institute commissioned a survey of teachers in K–8 elementary and middle schools in Common Core states to glean how much they've changed the way they teach—and whether they are seeing improvements in students' math achievement as a result.[6]

The present analysis is based on an online survey of a representative sample of 1,003 K–8 public school math teachers from the forty-three states (as well as the District of Columbia) that had adopted and retained the Common Core State Standards for Mathematics as of March 2015.[7] The excluded "non-adopting" states are Alaska, Indiana, Minnesota, Nebraska, Oklahoma, Texas, and Virginia. The teachers who participated in the survey worked with a range of students in a variety of geographic and academic settings. Most taught in schools where the CCSS-M had been in place for at least two years—and the vast majority (85 percent) report having also taught to other math standards, giving them a strong point of comparison.

The study sought to answer the following questions:

1. **To what extent does the content being taught reflect the CCSS-M?**
2. **In which ways are teachers changing their instructional practices to implement the CCSS-M?**
3. **What impact do teachers think the CCSS-M are having on students' mathematical preparation?**

Findings

Question 1: To what extent does the content being taught reflect the CCSS-M?

- *In general, teachers say that they are teaching the grade-level topics delineated in the CCSS-M.*

  Across all grades, thirty-seven of the forty-four "major" topics included in the survey were identified by at least 90 percent of teachers (from the appropriate grades) as among those they teach.

- *Most teachers are not neglecting computation, though many report having fewer students who memorize basic math formulas or multiplication tables.*

  For instance, in terms of computation, at least 90 percent of teachers report that they teach their students to "use place value understanding and properties of operations to add and subtract" (grade 2) and "solve problems involving the four operations, and identify and explain patterns in arithmetic" (grade 3). Furthermore, 29 percent of K–2 teachers report making computation a higher priority under CCSS-M—which makes sense given its stronger focus on math facts in the early grades.

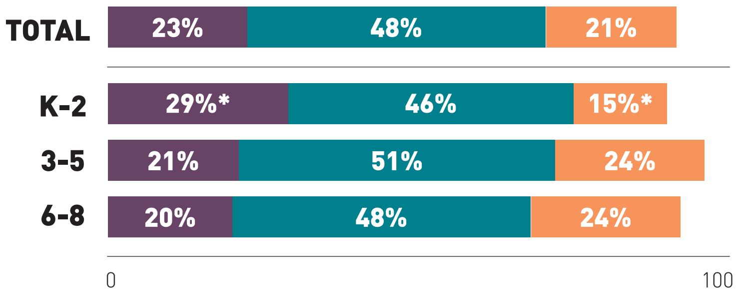
  Yet teachers in each grade band—and especially at the middle school level—are more likely to say that they have fewer students who memorize basic math formulas and times tables. The CCSS-M specify that by the end of grade 2, students will know from memory their addition facts; by the end of grade 3, they will know from memory their multiplication facts. As memorization is one strategy for doing so, it is unclear why more teachers report that fewer students are memorizing.

Table ES-1 • Little change in the priority given to computation

Q *Compared to before the Common Core math standards were implemented, are you now doing more, about the same, or less of each of the following in your classroom?*

More
About the same
Less

Prioritizing computation



* Indicates statistically significant difference at the 95 percent confidence level when comparing grade bands. For instance, this table shows that K–2 teachers are more likely than teachers in the later grades to be doing "more" prioritizing of computation—and less likely to be doing "less."

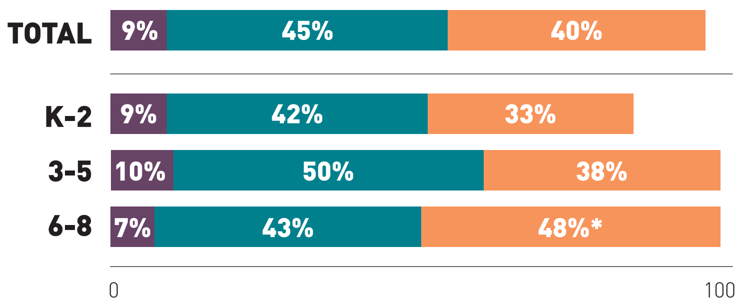
Note: Question wording in the tables throughout this report may be slightly edited for space. Column percentages may not total to 100 percent due to rounding or the omission of answer categories (e.g., "not sure" or "not applicable"). See Appendix B for complete question wording and data for all answer categories.

Table ES-2 • Fewer students are memorizing

Q *Compared to before the CCSS-M were implemented, do you think you now have more, about the same number, or fewer students who memorize basic math formulas or times-tables?*

More
About the same
Fewer

Students who memorize basic math formulas or times tables



* Indicates statistically significant difference at the 95 percent confidence level when comparing grade bands. For instance, this table shows that teachers in grades 6–8 are more likely than teachers in the earlier grades to have "fewer" students who memorize basic math formulas or times tables.

- *Teachers are paying more attention to the application of mathematics.*

  The standards expect students to use math in situations that require mathematical knowledge. At every grade level, application-related topics are taught by nearly all teachers. Further, many of the topics that teachers report spending the most time on are also related to application.

- *Some teachers say that their curricula and instructional materials are not well aligned with the CCSS-M.*[8]

  Data show 42 percent reporting that the math materials available to them are not well aligned with the CCSS-M, though a slight majority (55 percent) disagree.

- *Most teachers are modifying the pace of their curricula because the needs of their students demand it.*

  Two-thirds report that they "often modify" the pace of their curricula. Fifty-five percent often modify when it comes to deciding which math topics to cover, and approximately one-third modify when it comes to deciding the order of math topics.

Question 2: In which ways are teachers changing their instructional practices to implement the CCSS-M?

Overall, data show that teachers are changing their instructional practices in three key ways.

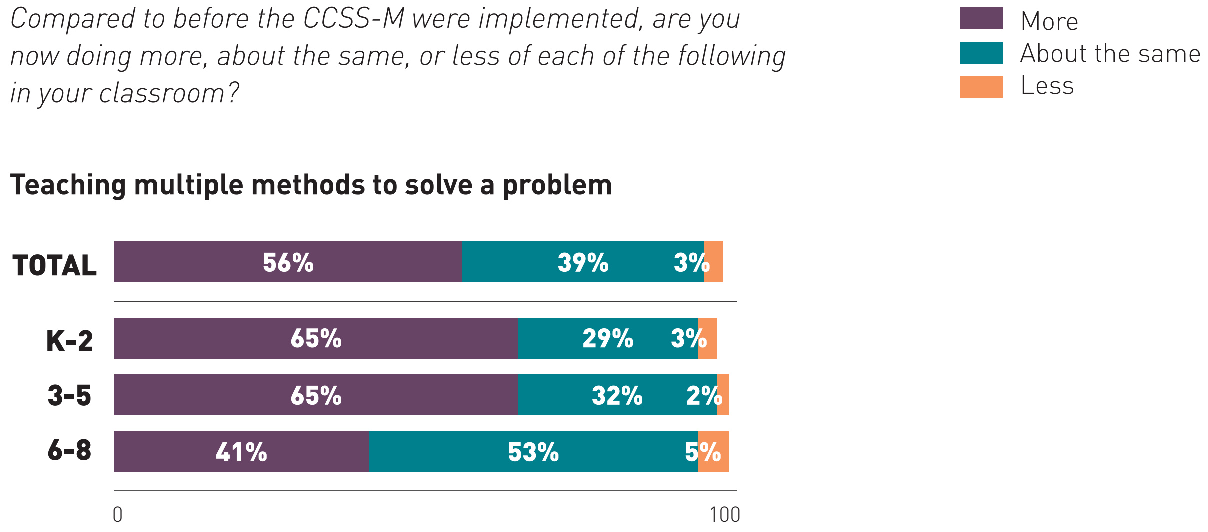
- *More teachers are teaching students multiple methods to solve problems.*

  Consistent with the Common Core expectation that students be able to "access concepts from a number of perspectives," 65 percent of both K–2 and 3–5 teachers and 41 percent of 6–8 teachers report that they are "teaching multiple methods to solve a problem" more often than they did before the CCSS-M were implemented; just 2–5 percent at all grade bands report doing this less frequently.

  Many teachers report challenges with this shift, however; 53 percent overall agree that "students are frustrated because they are being asked to learn many different ways to solve the same problem."

Table ES-3 • More teachers are teaching students multiple methods to solve a problem

Q Compared to before the CCSS-M were implemented, are you now doing more, about the same, or less of each of the following in your classroom?



- *More teachers are requiring students to use writing to explain their thinking.*

  The Standards for Mathematical Practice state that mathematically proficient students "construct viable arguments and critique the reasoning of others."[9] Across grades, teachers report a greater focus on verbal reasoning and the use of language since the CCSS-M were introduced. For instance, 64 percent of teachers say that they are more often "requiring students to explain in writing how they got their answers," while just 3 percent say that they are doing so less frequently.

  Interestingly, familiarity with the CCSS-M is associated with greater shifts in practice. For instance, teachers who have been teaching to the CCSS-M for longer (four years) are more likely than those who have taught to them for a shorter period (one year) to require that students explain in writing how they got their answers (67 percent versus 53 percent). Similar disparities appear relative to requiring the use of proper math vocabulary, teaching multiple methods, and teaching using the number line.

- *Overall, teachers are changing many of their practices in tune with the CCSS-M; other changes that they are making don't appear in the CCSS-M at all.*

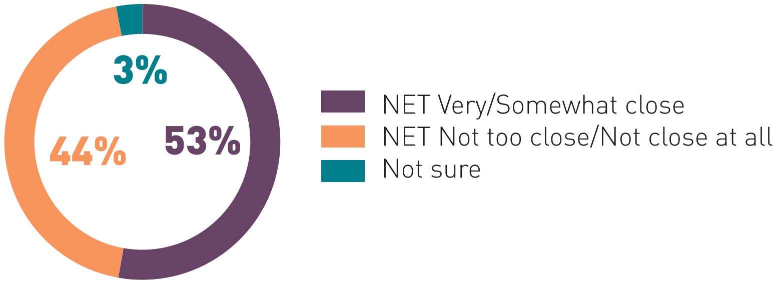
  Overall, teachers' responses suggest that they are changing many of their math practices in ways that are consistent with the CCSS-M. For example, 37 percent say that they are incorporating more teaching with the number line, which is consistent with a host of standards in grades 2–8.

  Yet in some cases, there is a murky connection between what the standards say and how teachers are implementing them. For example, 32 percent report that they are "using games and other student-directed activities" more, though there is nothing in the standards suggesting such a shift. Conversely, 40 percent say that they are using "flash cards and drills" less, though nothing in the standards discourages the use of these tools (especially since they are consistent with the expectation that students "know from memory" their multiplication tables by grade 3).

Table ES-4 • Frustration is high among students who are being asked to learn many different ways to solve the same problem

Q *How close does each of the following statements come to your view on the impact of the CCSS-M in the classroom?*

Students are frustrated because they're being asked to learn many different ways to solve the same problem



Question 3: What impact do teachers think the CCSS-M are having on students' mathematical preparation?

The survey yielded a mixed bag of findings when it comes to how teachers perceive the CCSS-M impacting their students' mathematical preparation.

- *Teachers think students are developing better number sense.*

  Seventy-seven percent of K–2 teachers, and majorities of teachers in grades 3–5 and 6–8, agree that "students are developing a stronger number sense and more ability to apply math in real-world situations." This is a positive result that likely reflects the new standards' increased emphasis on conceptual understanding and application.

- *Teachers are divided over students' ability to perform "simple calculations."*

  Consistent with the expectation in CCSS-M that students be fluent in the standard algorithm for each of the four basic operations, 32 percent of K–2 teachers say that they have more students who can "do simple calculations with speed and accuracy" now than before the CCSS-M (22 percent say fewer). This is reversed, however, in the other two grade bands, with larger numbers of teachers reporting that fewer students can complete simple calculations. The results for middle school teachers are particularly concerning, with just 13 percent reporting that more students can perform simple calculations and 39 percent reporting that fewer can. (Note that these middle school students started elementary school before the Common Core standards were adopted and implemented.)

- *Teachers with students who are below grade level have a more negative view of the impact of the standards.*

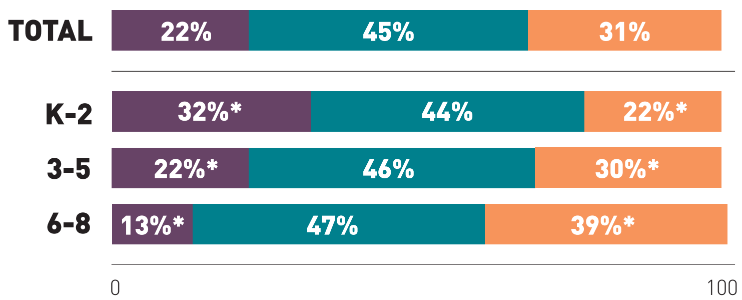
  The 18 percent of respondents who primarily teach students who are remedial or significantly below grade level in math evince significantly more pessimism about the impact of CCSS-M on students than teachers who primarily teach on-grade-level students. For instance, the former are less likely to agree that under CCSS-M, "students are developing a stronger number sense and more ability to apply math in real-world situations" (56 percent versus 66 percent) and more likely to report that fewer students "are able to do simple calculations with speed and accuracy" (44 percent versus 28 percent).

Table ES-5 • Teachers are divided over students' ability to do simple calculations

Q *Compared to before the CCSS-M were implemented, do you think you now have more, about the same number, or fewer of the following kinds of students in your classroom?*

More
About the same
Fewer

Students who are able to do simple calculations with speed and accuracy



* Indicates statistically significant difference at the 95 percent confidence level when comparing grade bands.

- *Teachers think the new standards are stressful for students.*

  In general, teachers see the CCSS-M as a source of stress for students. For instance, 42 percent of teachers overall say that they have more students with "math anxiety" than before the CCSS-M were implemented, and 53 percent agree that "expectations are unrealistic." In each of these cases, the higher the grade band, the more likely teachers are to report that students are encountering difficulties. (Again, many of these teachers work with students who started elementary school in the pre-Common Core era.)

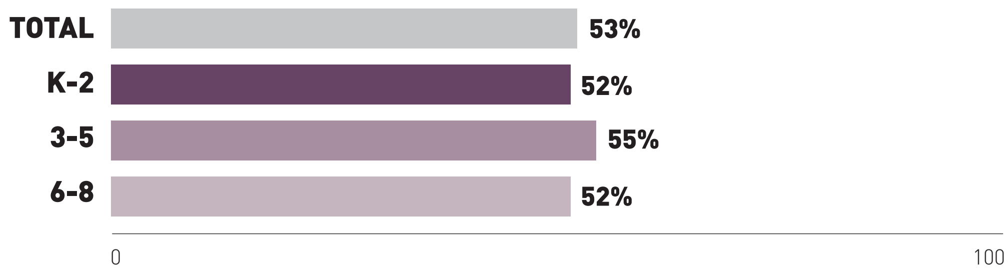
- *A majority of teachers think the CCSS-M will have long-term benefits for students.*

  A majority of teachers overall (53 percent) report that the statement "Students are getting better prepared for the advanced math needed to succeed in selective colleges or as STEM majors" is very or somewhat close to their view; 34 percent say that it is not close, and 14 percent are not sure. Similarly, a majority (55 percent) report that the statement "The standards will help ensure that America's young people have the math skills needed to compete in the global economy" is very or somewhat close to their view; 36 percent say that it is not close, and 9 percent are not sure.

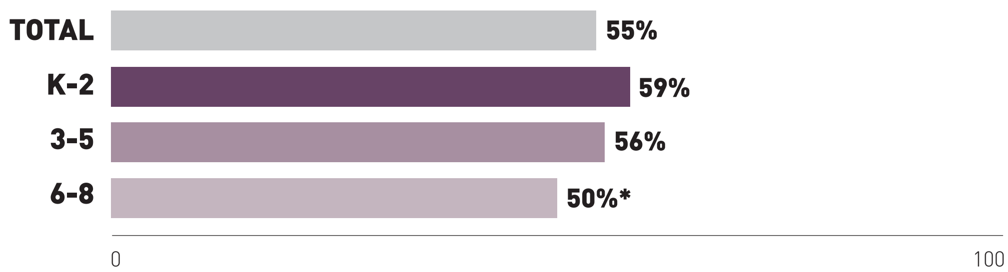
Table ES-6 • A majority of teachers think the CCSS-M will have long-term benefits for students

Q *How close does each of the following statements come to your view on the broader impact of the CCSS-M beyond the classroom? (Very or somewhat close to my view)*

Students are getting better prepared for the advanced math needed to succeed in selective colleges or as STEM majors



The standards will help ensure that America's young people have the math skills needed to compete in the global economy



* Indicates statistically significant difference at the 95 percent confidence level when comparing grade bands.

In sum, America's math teachers appear to be embracing the shifts articulated in the Common Core math standards and are, on the whole, optimistic that they will better prepare their students for advanced math and the changing economy. But challenges are apparent, including a lack of aligned curricular materials for some and the rather negative view about CCSS-M's impact held by teachers of below-grade-level students. Addressing these challenges will be imperative if the potential of the Common Core initiative is to be realized.

Introduction

Successful implementation of the Common Core State Standards for Mathematics (CCSS-M) should result in noticeable differences in primary and middle school math classrooms across the United States. After all, compared to most of the state standards they replaced, the CCSS-M focus on fewer topics, link concepts across grades more effectively, and increase attention to rigor.[10] In turn, these "shifts" are intended to support more focused, coherent, and rigorous instruction. But are the intended and expected changes actually occurring at the classroom level?

To find out, the Thomas B. Fordham Institute commissioned a survey of elementary and middle school math teachers in Common Core states to find out how much they've changed the way they teach—and whether they are seeing changes in students' math achievement as a result.

The study sought to answer the following questions:

1. **To what extent does the content being taught reflect the CCSS-M?**
2. **In which ways are teachers changing their instructional practices to implement the CCSS-M?**
3. **What impact do teachers think the CCSS-M are having on students' mathematical preparation?**

This report is organized into three sections. Section One provides a brief overview of CCSS-M and the survey. Section Two provides a detailed look at the findings. Section Three is a discussion of the findings and takeaways. Appendices A and B contain a full description of the methodology and the complete survey results, respectively.

Overview of the CCSS-M

The CCSS-M were released in June 2010, and by 2013, forty-five states and the District of Columbia had adopted them as their own. They were designed to address the widespread dissatisfaction among American educators with math standards that were "a mile wide and an inch deep" and that needed to become significantly more focused, coherent, and rigorous in order for math achievement to improve—a long-sought goal for both educators and policymakers.[11] To that end, the CCSS-M include two types of standards:

- The Standards for Mathematical Content, which are specific to each grade from Kindergarten to grade 8;[12] and
- The Standards for Mathematical Practice, which describe the "processes and proficiencies" associated with mathematical competence at all grade levels (see *Standards for Mathematical Practice*).

Both sets of standards reflect three "shifts" in mathematics related to *focus*, *coherence*, and *rigor* (see *Key Shifts in the CCSS-M*).

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

What Does the Literature Say about CCSS-M Implementation?

Most of the extant research on CCSS-M implementation is based on survey data gleaned from teachers and district leaders. Broadly speaking, it has focused on two questions: How are teachers implementing the standards? And how are districts supporting that implementation?

Overall, the evidence is mixed that teachers are implementing the standards with fidelity. For example, in 2013 Dingman et al. compared previous K–8 state math standards to the CCSS-M and identified four key changes: the timing of content delivery, the frequency of particular mathematic topics across grades, the emphasis placed on certain topics, and "the nature and level of reasoning expectations."[13] Yet in that same year, Cogan et al. found that 77 percent of math teachers said the CCSS-M were "pretty much the same" as their previous state mathematics standards.[14]

Similarly, in a 2013 survey by Davis et al., just 40 percent of teachers said the CCSS-M would require them to increase their emphasis on conceptual understanding (one of three key shifts that are fundamental to the standards).[15] However, in a 2016 study by Kane et al., 81 percent of math teachers said they had indeed increased their emphasis on conceptual understanding, and 78 percent said they had increased their emphasis on real-world application.[16] A majority of math teachers in a recent RAND Corporation survey also reported encouraging their students to use mathematical language and symbols appropriately and to explain and justify work—both of which are encouraged in the CCSS-M.[17]

Overall, the evidence suggests that districts have struggled to support teachers during the transition to the CCSS-M, particularly with regard to instructional materials and providing time to collaborate. For instance, only one-third of the teachers in the RAND Corporation's study indicated that their main instructional materials addressed the three aspects of rigor delineated in the CCSS-M with equal time and intensity. In 2012, the Education Research Center reported that 89 percent of teachers indicated that planning time with colleagues was the most helpful form of CCSS professional development.[18] Yet three years later, just one-third of the teachers in the Kane study said they had collaborated on aligning materials and assessments with the CCSS.

As these findings suggest, CCSS-M implementation remains a work in progress. Like prior studies, this one examines the topics that teachers are teaching as well as changes in their instructional practices. Yet the survey's design and use of focus groups also provide a richer and more nuanced understanding of the struggles that teachers face in implementing the CCSS-M. In particular, our data show how the tensions between conceptual understanding, procedural skill and fluency, and application create real challenges for teachers seeking to promote rigor.

The Survey

This study is based on an online survey of a representative sample of 1,003 K–8 public school math teachers[19] from the forty-three states plus the District of Columbia that had adopted (and retained) the Common Core State Standards for Mathematics as of March 2015.[20] The excluded "non-adopting" states are Alaska, Indiana, Minnesota, Nebraska, Oklahoma, Texas, and Virginia. The teachers who participated in the survey worked with a range of students in a variety of geographic and academic settings. Most taught in schools where the CCSS-M had been in place for at least two years—and the vast majority (85 percent) report having also taught to other math standards, giving them a strong point of comparison (see Table 1).

Key Shifts in the CCSS-M

Focus: Greater focus on fewer topics.

Rather than racing to cover topics in a superficial way, the CCSS-M expect teachers to significantly narrow and deepen the way time and energy are spent in the math classroom. The standards focus deeply on the "major work" of each grade, or those topics that are deemed essential to college and career readiness and thus merit greater attention. In particular, the K–8 standards focus on the progression from arithmetic to algebra.[21]

Coherence: Linking topics and thinking across grades.

Mathematics is a coherent body of knowledge made up of interconnected concepts. Under the standards, learning is carefully connected across grades so that students can build new understanding onto foundations built in previous years. Each standard is not a new event, but an extension of previous learning.

Rigor: Pursue conceptual understanding, procedural skills and fluency, and application with equal intensity.

Rigor refers to deep, authentic command of mathematical concepts, not making math harder or introducing topics at earlier grades. The standards require that educators pursue three aspects of rigor with equal intensity: conceptual understanding, procedural skills and fluency, and application.

1. *Conceptual understanding*: Students access concepts from a number of perspectives so that they are able to see math as more than a set of discrete procedures.
2. *Procedural skill and fluency*: Students compute with speed and accuracy.

The survey was fielded between March 30 and May 15, 2015, and was preceded by three focus groups, each of which included teachers from a different grade band and region (K–2 in Baltimore, MD; 3–5 in Louisville, KY; and 6–8 in Walnut Creek, CA). Data from the focus groups informed the design of the survey instrument, which was pre-tested and programmed for administration online. Illustrative quotes from focus group participants and open-ended responses from survey participants are presented throughout.

A complete discussion of the methodology can be found in Appendix A.

Table 1 • Characteristics of the sample

Metro Status	%
Urban	26
Rural	35
Suburban	39

Grade Band	%
K–2	32
3–5	33
6–8	35

Region	%
Northeast	25
Midwest	22
South	31
West	22

Students Eligible for Free or Reduced-Price Lunch	%
0–24 %	18
25–49 %	24
50–74 %	26
75–100 %	32

Experience Teaching the CCSS-M	%
Common Core only	15
Also other standards	85

Teacher Gender	%
Male	13
Female	88

Year CCSS-M Implemented at School	%
2011	15
2012	30
2013	38
2014	16
Not implemented	2

Key Shifts in the CCSS-M, (Continued)

3. *Application*: Students use math in situations that require mathematical knowledge, which requires conceptual understanding and procedural fluency.

The relative importance of these three aspects of rigor has been the subject of intense debate within the math field, and the emphasis that each receives largely depends on the teacher and curriculum. As explained in an ancillary CCSS-M document:[22]

Some curricula stress fluency in computation without acknowledging the role of conceptual understanding in attaining fluency and making algorithms more learnable. Some stress conceptual understanding without acknowledging that fluency requires separate classroom work of a different nature. Some stress pure mathematics without acknowledging that applications can be highly motivating for students and that a mathematical education should make students fit for more than just their next mathematics course. At another extreme, some curricula focus on applications without acknowledging that math doesn't teach itself.

The CCSS-M attempts to strike a solid, durable balance among these three aspects without sacrificing any of them: "The Standards do not take sides… rather they set high expectations for all three components of rigor in the major work of each grade."[23]

Adapted from Common Core State Standards Initiative's "Key Shifts in Mathematics," http://www.corestandards.org/other-resources/key-shifts-in-mathematics/.

Findings

Question 1: To What Extent Does the Content Being Taught Reflect the CCSS-M?

- *In general, teachers say that they are teaching the grade-level topics delineated in the CCSS-M.*
- *Most teachers are not neglecting computation, though many report having fewer students who memorize basic math formulas or multiplication tables.*
- *Teachers are paying more attention to the application of mathematics.*
- *Some teachers say their curricula and instructional materials are not well aligned with the CCSS-M.*
- *Most teachers are modifying the pace of their curricula because the needs of their students demand it.*

In general, teachers say that they are teaching the grade-level topics delineated in the CCSS-M.

For each grade (K–8) the CCSS-M include twenty-two to twenty-nine grade-level standards, which are grouped into eight to twelve "clusters." Clusters are groups of related standards that are categorized in ancillary documents as *major*, *supporting*, or *additional*, reflecting the emphasis the standards place on each.[24] To gauge the extent to which they are teaching the appropriate grade-level clusters (also referred to as "topics"), teachers in the survey were shown a list of fourteen clusters,[25] including four to six major and two to four supporting or additional clusters from their grade level,[26] and six decoy clusters from grades above or below their own.[27] Teachers were then asked to identify the clusters they would teach by the end of the school year.

At each grade level, the major grade-level clusters were identified by a clear majority of teachers as among those they would teach (see Table 2).[28] For example, 98 percent of Kindergarten teachers said they would teach students to compare numbers. Across all grades, thirty-seven of the forty-four major topics that were included in the survey were identified by at least 90 percent of teachers from the appropriate grade as among those they would teach.

Table 2 • In general, teachers are covering the major grade-level clusters

Kindergarten (n=103)		Will teach this cluster
Domain	**Major Cluster**	**Percent**
Counting and Cardinality	Compare numbers	98
Counting and Cardinality	Count to tell the number of objects	95
Counting and Cardinality	Know number names and the count sequence	95
Operations and Algebraic Thinking	Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from	95
Operations and Algebraic Thinking	Work with numbers 11–19 to gain foundations for place value	92

Grade 1 (n=104)		Will teach this cluster
Domain	**Major Cluster**	**Percent**
Operations and Algebraic Thinking	Add and subtract within 20	96
Operations and Algebraic Thinking	Represent and solve problems involving addition and subtraction	92
Number and Operations in Base 10	Use place value understanding and properties of operations to add and subtract	90
Number and Operations in Base 10	Extending the counting sequence	83
Measurement and Data	Measure lengths indirectly and by iterating length units	77

Grade 2 (n=108)		Will teach this cluster
Domain	**Major Cluster**	**Percent**
Number and Operations in Base 10	Use place value understanding and properties of operations to add and subtract	98
Operations and Algebraic Thinking	Add and subtract within 20	95
Operations and Algebraic Thinking	Represent and solve problems involving addition and subtraction	95
Measurement and Data	Measure and estimate lengths in standard units	90

Grade 3 (n=108)		Will teach this cluster
Domain	**Major Cluster**	**Percent**
Operations and Algebraic Thinking	Understand properties of multiplication and the relationship between multiplication and division	96
Operations and Algebraic Thinking	Represent and solve problems involving multiplication and division	95
Measurement and Data	Geometric measurement: understand concepts of area and relate area to multiplication and to addition	95
Number and Operations – Fractions	Develop understanding of fractions as numbers	94
Operations and Algebraic Thinking	Solve problems involving the four operations, and identify and explain patterns in arithmetic	90
Measurement and Data	Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects	81

Grade 4 (n=116)		Will teach this cluster
Domain	**Major Cluster**	**Percent**
Operations and Algebraic Thinking	Use the four operations with whole numbers to solve problems	99
Number and Operations in Base 10	Generalize place value understanding for multi-digit whole numbers	97
Number and Operations – Fractions	Extend understanding of fraction equivalence and ordering	96
Number and Operations – Fractions	Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers	93
Number and Operations – Fractions	Understand decimal notation for fractions, and compare decimal fractions	93

Grade 5 (n=100)		Will teach this cluster
Domain	**Major Cluster**	**Percent**
Number and Operations in Base 10	Understand the place value system	100
Number and Operations – Fractions	Use equivalent fractions as a strategy to add and subtract fractions	100
Number and Operations in Base 10	Perform operations with multi-digit whole numbers and with decimals to hundredths	99
Measurement and Data	Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition	93
Number and Operations – Fractions	Apply and extend previous understandings of multiplication and division to multiply and divide fractions	92

Grade 6 (n=110)		Will teach this cluster
Domain	**Major Cluster**	**Percent**
The Number System	Apply and extend previous understandings of multiplication and division to divide fractions by fractions	92
Expressions and Equations	Reason about and solve one-variable equations and inequalities	91
Ratios and Proportional Relationships	Understand ratio concepts and use ratio reasoning to solve problems	90
Expressions and Equations	Apply and extend previous understandings of numbers to the system of rational numbers	89
Expressions and Equations	Represent and analyze quantitative relationships between dependent and independent variables	74

Grade 7 (n=120)

Domain	Major Cluster	Will teach this cluster Percent
Ratios and Proportional Relationships	Analyze proportional relationships and use them to solve real-world and mathematical problems	94
Expressions and Equations	Solve real-life and mathematical problems using numerical and algebraic expressions and equations	93
The Number System	Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers	92
Expressions and Equations	Use properties of operations to generate equivalent expressions	90

Grade 8 (n=121)

Domain	Major Cluster	Will teach this cluster Percent
Expressions and Equations	Analyze and solve linear equations and pairs of simultaneous linear equations	96
Geometry	Understand and apply the Pythagorean Theorem	94
Functions	Define, evaluate, and compare functions	90
Expressions and Equations	Work with radicals and integer exponents	86
Geometry	Understand congruence and similarity using physical models, transparencies, or geometry software	72

Note: Domains and clusters are larger and smaller groups of related standards, respectively. Topics that are off grade level are not included. Shaded rows indicate topics that less than 90 percent of teachers in the appropriate grade level will teach.

The survey also asked teachers to identify up to five topics on which they would "spend the most amount of time" during the school year (not shown).[29] These data suggest a few findings:[30]

- Grades K–2 teachers are spending most of their time on numeration and arithmetic, with a majority identifying the following clusters as among their top five time commitments: "Count to tell the number of objects" (52 percent of Kindergarten teachers); "Add and subtract within 20" (86 percent of grade 1 teachers); and "Represent and solve problems involving addition and subtraction" (75 percent of grade 2 teachers).

- Grades 6–7 teachers are emphasizing ratios and proportional relationships (a top-five cluster for 66 percent of grade 6 teachers and 77 percent of grade 7 teachers). CCSS-M uses Ratios and Proportional Relationships to organize much of the work that occurs in grades 6–7.

- Grades 6–8 teachers are focusing on the algebra expectations of the standards (expressions and equations), with strong attention in grade 8 to linear equations and pairs of simultaneous linear equations (a top-five cluster for 88 percent).

- Grade 8 teachers may not be spending enough time on congruence and similarity, which only 17 percent of the teachers identify as a top-five cluster.

Most teachers are not neglecting computation, though many report having fewer students who memorize basic math formulas or multiplication tables.

Large majorities of educators report teaching clusters related to basic operations and computation. For example, at least 90 percent of teachers say they teach the following grade-level clusters:

- Use place value understanding and properties of operations to add and subtract (grades 1 and 2);
- Solve problems involving the four operations, and identify and explain patterns in arithmetic (grade 3);
- Use the four operations with whole numbers to solve problems (grade 4);
- Perform operations with multi-digit whole numbers and with decimals to hundredths (grade 5); and
- Compute fluently with multi-digit numbers and find common factors and multiples (grade 6).[31]

When asked directly about prioritizing computation, 23 percent of teachers overall say they do it *more* now than before the CCSS-M were implemented; 21 percent say they do it *less*; and 48 percent say they do it *about the same* amount (Table 3). In K–2, where one would expect to see more emphasis on computation, 29 percent of teachers report giving it greater priority. In short, the data show that computation has *not* been neglected since the CCSS-M were implemented.

One strategy for teaching basic facts and computation in the early grades is memorization. Yet teachers in each grade band are more likely to say they have fewer students who memorize basic math formulas and times tables (Table 4). For example, 33 percent of K–2 teachers say they have fewer students who memorize; while just 9 percent say they have more. The CCSS-M specifies that students will know from memory their addition facts by the end of grade 2, and their multiplication facts by grade 3 ("know from memory all sums of two one-digit numbers" and "know from memory all products of two one-digit numbers," respectively).[32]

As others have explained,[33] "knowing from memory" is an outcome—as opposed to "memorization," which is one of many strategies to reach that outcome. (The standards themselves reference various "mental strategies"). If teachers understand that difference, it is possible that they are using other mental strategies (such as "make 10") to help students know their basic facts from memory. Alternatively, they may think the CCSS-M discourage memorization though they do not. Or perhaps increased attention to applications has simply meant less time or need for memorization.

In Their Own Words: Still Too Much Content

Two out of three teachers surveyed do *not* believe that "important math concepts are being missed because fewer topics are being covered." Yet several suggested in the open-ended comments that there was *still* too much content to cover.

➔ *It still seems that we are rushing kids through a math curriculum.* **—Grade 7, Kansas**

➔ *The Common Core Standards are great in theory, but by cramming so much into each grade teachers are forced to move quickly from topic to topic regardless [of whether] students have mastered the previous topic.* **—Grade 6, North Carolina**

➔ *I think there are way too many standards to teach to go into depth on any one of them. I feel that I am only introducing my students to the concepts and not spending enough time on any of them. If Common Core was supposed to lessen the number of standards taught, it sure doesn't seem that way to me.* **—Grade 6, Florida**

➔ *The students are not being given enough time to learn the concepts being taught. Instead of teaching fewer concepts more in depth, we are expected to teach many concepts too fast, not giving the children enough time to master anything.* **—Grade 1, Florida**

➔ *While I believe that narrowing the curriculum to focus on important standards WOULD actually improve teachers' ability to go more in-depth and deepen student understanding, I don't think the Common Core State Standards ACTUALLY do this. Within "one" standard, there are often MANY learning targets that may require entire units to cover!* **—Grade 8, New York**

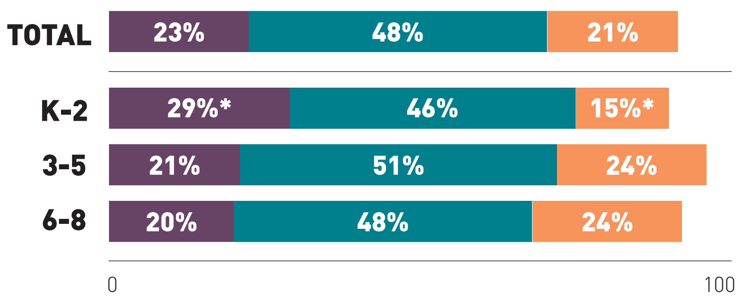
Table 3 • Little change in the priority given to computation

Q | *Compared to before the Common Core math standards were implemented, are you now doing more, about the same, or less of each of the following in your classroom?*

More
About the same
Less

* Indicates statistically significant difference at the 95 percent confidence level when comparing grade bands.

Prioritizing computation



Note: Question wording in the tables throughout this report may be slightly edited for space. Column percentages may not total to 100 percent due to rounding or the omission of answer categories (e.g., "not sure" or "not applicable"). See Appendix B for complete question wording and data for all answer categories.

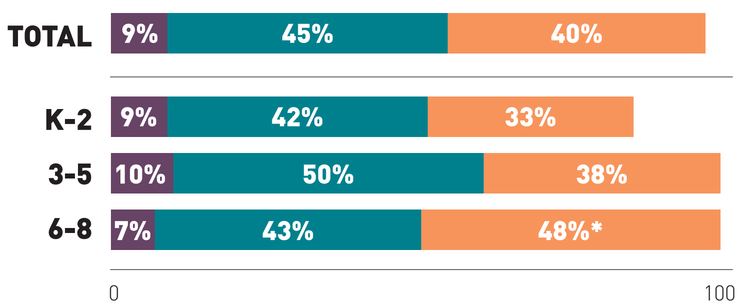
Table 4 • Fewer students are memorizing

Q | *Compared to before the CCSS-M were implemented, do you think you now have more, about the same number, or fewer students who memorize basic math formulas or times-tables?*

More
About the same
Fewer

* Indicates statistically significant difference at the 95 percent confidence level when comparing grade bands.

Students who memorize basic math formulas or times tables



Are Teachers Teaching Below-Grade-Level Content?

Reviewing content from the previous grade is one thing. But in some cases, a majority of teachers say they teach clusters from at least two grade levels below their own (Table 5).[34]

Table 5 • Below-grade-level clusters are taught by more than half of teachers

Grade level	Below-grade-level cluster	Percent of teachers who teach this cluster
2	Kindergarten: Classify objects and count the number of objects in categories	53
4	Grade 2: Measure and estimate lengths in standard units	88
5	Grade 3: Understand properties of multiplication and the relationship between multiplication and division	97
6	Grade 4: Use the four operations with whole numbers to solve problems	84
7	Grade 5: Use equivalent fractions as a strategy to add and subtract fractions	83
8	Grade 6: Solve real-world and mathematical problems involving area, surface area, and volume	74

The CCSS-M are meant to strengthen states' historically weak math standards, so it's troubling to see so many teachers teaching below-grade-level content.[35] Achieving the curricular coherence envisioned by the CCSS-M requires understanding *exactly* what one should be teaching and what should already have been taught.

Yet coherence also entails building on prior knowledge, which often means that teachers include content from earlier grades. Thus, it's possible that topics from earlier grades are being reviewed, or used as scaffolding for grade-level content, rather than re-taught. (It's also possible that students are simply behind, or have gaps in their knowledge as a consequence of the transition to the CCSS-M, leaving teachers little choice but to meet them where they are.) These nuances aren't captured by the survey. However, it's worth noting that 38 percent of teachers report doing more "linking new math concepts to those taught in earlier grades" than before the CCSS-M, while only 5 percent report doing less of this.

Teachers are paying more attention to the application of mathematics.

The materials associated with CCSS-M suggest that concepts, procedures, and applications be pursued with "equal intensity," and the terms "apply," "use," and "solve" appear multiple times in every grade's content standards.

Because of this shift, one would hope to see more attention being paid to application since the adoption of the standards, and there is good evidence that this is occurring. For example, 58 percent of teachers overall report that they are spending more time "teaching multi-step word problems" than before the CCSS-M were implemented,[36] and 23 percent report spending more time "integrating math concepts into other subjects" that they teach.

At every grade level, application-related topics are taught by nearly all teachers. Further, many of the topics teachers report spending the most time on are also related to application, as shown in Table 6. (See *Do Teachers Think Students Can Apply Their Knowledge?*)

Learning Gaps

As students transitioned from one set of standards to the other, gaps were created and they did not learn everything they needed. For example, third graders came into fourth grade not knowing their multiplication facts because it was not part of the old standard and that made fourth grade math more difficult for them. —**Grade 4, Florida**

Table 6 • Most teachers are teaching application

Major clusters in each grade level that involve "applying procedures."[37]	Percent of teachers who will teach topic
Grade 1: Represent and **solve problems** involving addition and subtraction (solve word problems)	92
Grade 2: Represent and **solve problems** involving addition and subtraction (one- and two-step word problems)	95
Grade 3: Represent and **solve problems** involving multiplication and division (solve word problems)	95
Grade 3: **Solve problems** involving the four operations, and identify and explain patterns in arithmetic (two-step word problems)	90
Grade 4: Use the four operations with whole numbers to **solve problems** (word problems and multi-step word problems)	99
Grade 5: **Apply** and extend previous understandings of multiplication and division to multiply and divide fractions (real-world problems)	92
Grade 6: Understand ratio concepts and **use** ratio reasoning to **solve problems** (real-world problems)	90
Grade 6: **Apply** and extend previous understandings of multiplication and division to divide fractions by fractions (word problems)	92
Grade 6: **Apply** and extend previous understandings of numbers to the system of rational numbers (real-world contexts; real-world and mathematical problems)	89
Grade 7: Analyze proportional relationships and use them to **solve real-world** and mathematical problems (multi-step problems)	94
Grade 7: **Apply** and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers (real-world and mathematical problems)	92

Note: Table lists the clusters (though in some cases it is actually the standards, which are noted in parentheses) that explicitly address word problems or real-world problems.

About Those "Other" Teachers...

Teachers express a high degree of confidence in their grade-level colleagues, with the vast majority describing teachers in their grade as having either excellent (47 percent) or good (43 percent) knowledge of Common Core topics. Yet confidence in their colleagues in neighboring grades is lower, with just 10 percent rating topic knowledge of those in the grade immediately below them "excellent," and just 11 percent assigning that rating to teachers in the grade immediately above them.

This lack of confidence in teachers from other grades could stem from any number of sources. For example, to the extent that teachers inherit students who have not mastered content from a previous grade, they may suspect their colleagues in that grade of instructional negligence. Alternatively, because they spend comparatively little time collaborating with colleagues from other grades, teachers may not know them well enough to have faith in their ability.

Do Teachers Think Students Can Apply Their Knowledge?

Although teachers are spending a great deal of time teaching students how to apply their knowledge, they have mixed views of students' progress on this front. For example, teachers are evenly divided over students' ability to "apply the correct math procedures in word problems without prompting from the teacher," with those in the higher grades expressing greater skepticism (Table 7). Similarly, when asked if they see more students "who can solve challenging problems" appropriate to their grade level, teachers are divided, with those at the K–2 level more likely to report progress.[38]

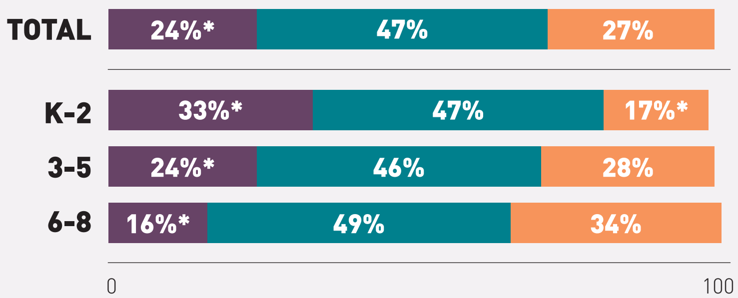
Table 7 • Views on students' application of knowledge

Q *Compared to before the CCSS-M were implemented, do you think you now have more, about the same number, or fewer of the following kinds of students in your classroom?*

More
About the same
Fewer

* Indicates statistically significant difference at the 95 percent confidence level when comparing grade bands.

Students who can apply the correct math procedures in word problems without prompting from the teacher



	K–2	3–5	6–8
Students who can solve challenging problems that require addition and subtraction and an understanding of place value			
More	37		
About the same	43		
Fewer	14		
Students who can solve challenging problems that require multiplication and division with whole numbers and fractions			
More		28	
About the same		46	
Fewer		24	
Students who understand and can solve challenging problems related to ratios and proportions			
More			27
About the same			45
Fewer			27

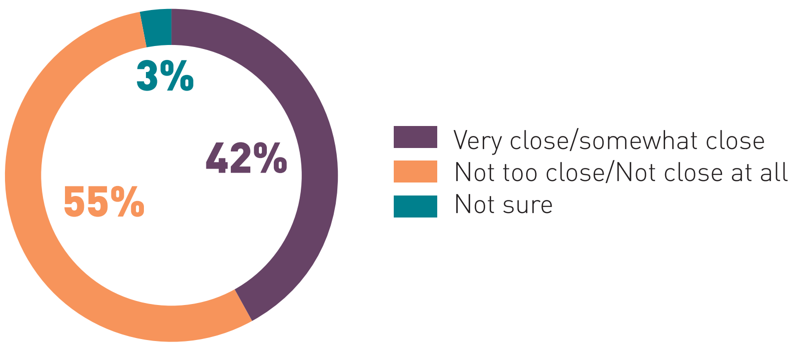
Some teachers say their curricula and instructional materials are not well aligned with the CCSS-M.

In a recent study, over 90 percent of districts in CCSS-adoption states reported that developing or identifying aligned materials has been a challenge,[39] and other studies suggest that many instructional materials remain poorly aligned.[40] Forty-two percent of teachers in our study report that the math materials available to them are *not* well aligned with the CCSS-M, though a slight majority (55 percent) disagrees (Table 8).

Table 8 • Over forty percent of teachers say math materials are not aligned to the CCSS-M

Q *How close does this statement come to your view on the impact of the CCSS-M in the classroom?*

The math materials available to me are not aligned with the standards.



In light of these findings, it's notable that 51 percent of teachers who have a primary textbook and 49 percent of those who are provided instructional materials say they are "required" to use them (Table 9).[41] Teachers in the lower grades are far more likely to be subject to these mandates than those in the higher grades.

Finally, although the CCSS-M are not a curriculum, they do have implications for what content math curricula should cover. Consequently, we might expect to see less variation in curricula in places where the standards have been implemented.

In Their Own Words: Materials Slow to Catch Up

Educators explain that instructional materials are still not where they need to be in terms of alignment to the CCSS-M:

- *I think the standards are awesome; the reason they aren't working here is because we are being told to follow a textbook curriculum that doesn't fit the standards.*
  —Grade 8, Utah
- *It would be great if I actually had curriculum that aligned to the Common Core and was complete with rubrics, assessments, workbooks, associated projects, etc. We are not curriculum writers yet no teacher across the nation has legitimate, worthwhile curriculum to help them teach the Common Core.*
  —Grade 6, Washington
- *Our school is trying to paste together our old curriculum and bridge materials plus a district pacing guide that was poorly written. It has been chaos.*
  —Grade 1, California
- *Common Core standards are not difficult to teach but there is not yet a good book which has problems that are aligned to them. The standards were adopted sooner than the book could get published. There are several publishers who just changed the appearance of their book and about 20 percent of their content and they are calling it a Common Core book.*
  —Grade 7, California

As shown in Table 10, strong majorities of teachers in each grade band say the math curriculum at their school is the same for all the classes in their grade; however, considerably fewer teachers say their schools use the same program across grades.[42] (See *Which Math Curricula Are Most Commonly Used?* for particular selections.)

Table 9 • Many teachers are required to use the textbooks and instructional materials they are provided, especially in the early grades

Thinking about the math curriculum used in your classroom this school year: (Yes)

	Total	K–2	3–5	6–8
Are you required to use the textbook, or not?	51	56	57	41*
Are you required to use the instructional materials, or not?	49	64*	52*	31*

* Indicates statistically significant difference at the 95 percent confidence level when comparing grade bands.

Note: Respondents are those who previously indicated they use a textbook and have been provided materials, respectively.

Table 10 • Most teachers say the math curriculum at their school is the same for all classes

Thinking about the math curriculum used in your classroom this school year: (Yes)

	Total	K–2	3–5	6–8
Is it the same for all math classes in your grade, or not?	82	92	90	65*
Is it the same for all grades throughout your school, or not?	61	67	69	49*

* Indicates statistically significant difference at the 95 percent confidence level when comparing grade bands.

Most teachers are modifying the pace of their curricula because the needs of their students demand it.

We asked teachers a series of questions about whether they modified particular aspects of their curriculum (Table 11). Two-thirds report that they "often modify" the *pace* of their curriculum. Fifty-five percent often modify when it comes to deciding which math *topics* to cover and approximately one-third modify when it comes to deciding the *order* of math topics. Vast majorities of teachers, who modify say they do so because the "learning needs of [their] students demand it," not because "the math department or district tells [them] to." However, some teachers may also be modifying because their textbooks are not well aligned with the CCSS-M, or to avoid creating gaps in learning during the transition from old to new standards.

Which Math Curricula Are Most Commonly Used?

Teachers were provided a list of titles and publishers and asked to select the "primary math curriculum" that they currently use in their classroom. The most frequently selected curricula in each grade band are shown below. (See Appendix B for the full list of primary math curricula shown in the survey.)

Grades K–2

Envision Math	16%
Go Math	15%
Everyday Mathematics	11%
EngageNY	10%

Grades 3–5

Go Math	16%
Envision Math	15%
EngageNY	10%

Grades 6–8

Math Connects	11%
Big Ideas Learning	9%
Holt McDougal Math	8%

Table 11 • Most teachers often modify the pacing and content of their curriculum, but not the sequencing

Generally speaking, when it comes to deciding the order of math topics (sequence), do you:

- **35%** Often modify the curriculum and instructional materials
- **64%** Tend to follow the curriculum and instructional materials as is

Generally speaking, when it comes to deciding the amount of time to allot to each math topic (pacing), do you:

- **67%** Often modify the curriculum and instructional materials
- **32%** Tend to follow the curriculum and instructional materials as is

Generally speaking, when it comes to deciding which math topics to cover (content), do you:

- **55%** Often modify the curriculum and instructional materials
- **44%** Tend to follow the curriculum and instructional materials as is

Question 2: In Which Ways Are Teachers Changing Their Instructional Practices to Implement the CCSS-M?

- *More teachers are teaching students multiple methods to solve problems.*
- *More teachers are requiring students to use writing to explain their thinking.*
- *Overall, teachers are changing many of their practices in tune with the CCSS-M; other changes that they are making don't appear in the CCSS-M at all.*

More teachers are teaching students multiple methods to solve problems.

Consistent with the expectation that students be able to "access concepts from a number of perspectives" (see pages 17–18, *Key Shifts in the CCSS-M*) and "check their answers to problems using a different method" (see page 16, *Standards for Mathematical Practice*), 65 percent of K–5 teachers and 41 percent of 6–8 teachers report that they are "teaching multiple methods to solve a problem" more often than they did before the CCSS-M were implemented. Just 2–5 percent at all grade bands report doing this less frequently. Many teachers appear to have reservations about this shift, however, with 53 percent overall agreeing that "students are frustrated because they are being asked to learn many different ways to solve the same problem."

Multiple Strategies = Multiple Woes?

I like Common Core, but find that my students get more confused when I introduce more strategies for solving problems. —Grade 2, Georgia

Has Collaboration between Teachers Increased Since the CCSS-M Were Adopted?

In a word, yes. Teachers are spending more time collaborating with their colleagues (especially those in the same grade level).

In fact, 52 percent of K–5 and 57 percent of middle school teachers say they are spending more time discussing mathematics curriculum and instruction with teachers *at their grade level*; while just 5 percent and 4 percent, respectively, report spending less time (see Table 12).

On the other hand, just 28 percent of K–5 and 37 percent of middle school teachers report spending more time discussing mathematics curriculum and instruction with teachers *in other grades*. In fact, at least one-fifth of elementary teachers say that discussions about math with teachers at other grade levels simply "doesn't happen." In other words, when it comes to collaborating in order to connect learning from one grade to the next, there is considerable room for improvement.

Table 12: Discussions among teachers in the same grade level are up

Compared to before the CCSS-M were implemented, are you now doing more, about the same, or less discussing of the math curriculum and instructional practices with the following teachers?

	More	About same	Less	Doesn't happen
Grades K–5				
With teachers in your grade	52	40	5	2
With teachers in other grades	28	44	5	20
Grades 6–8				
With math teachers in your grade	57	34	4	3
With math teachers in other grades	37	45	8	8

More teachers are requiring students to use writing to explain their thinking.

Consistent with the expectation that mathematically proficient students "construct viable arguments and critique the reasoning of others," teachers at all grade levels report a greater focus on verbal reasoning and the use of language since the CCSS-M were introduced. Fifty-five percent of teachers say they are devoting more attention to "requiring students to use proper math vocabulary," while just 2 percent report paying less attention to it. Similarly, 64 percent of teachers say they are "requiring students to explain in writing how they got their answers" more often, while just 3 percent say they are doing so less frequently (see Table 13). Overall, teachers appear to support these changes, with 66 percent essentially disagreeing that "requiring written explanation for simple math problems is unnecessary and detracts from real learning" (not shown).

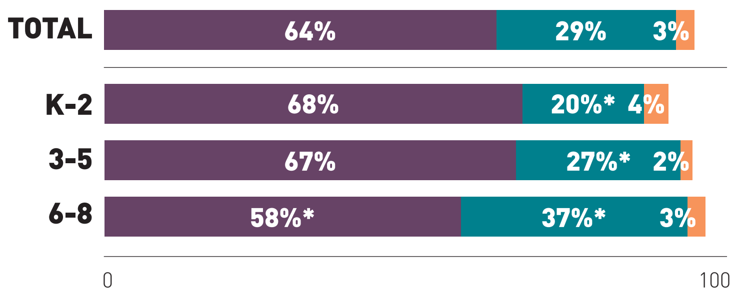
Table 13 • Use of language in the math classroom

Q *Compared to before the CCSS-M were implemented, are you now doing more, about the same, or less of each of the following in your classroom?*

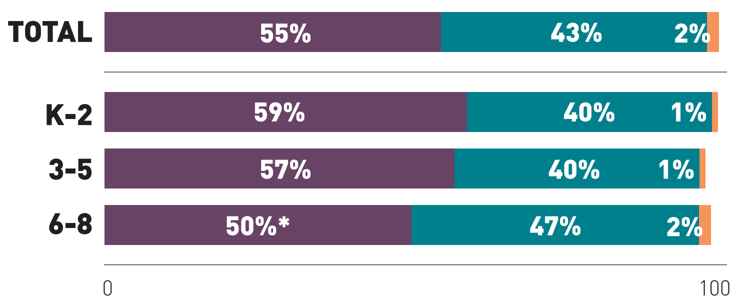
More
About the same
Less

* Indicates statistically significant difference at the 95 percent confidence level when comparing grade bands.

Requiring students to explain in writing how they got their answers



Requiring students to use proper math vocabulary



Overall, teachers are changing many of their practices in tune with the CCSS-M; other changes that they are making don't appear in the CCSS-M at all.

Teachers' responses suggest they are changing most of their math practice in ways that are consistent with the CCSS-M. For example, 37 percent say that they are incorporating more teaching with the number line, which is consistent with a host of standards in grades 2–8 (Table 14).[43] Yet in some cases, the connection between what the standards say and how teachers are implementing them is murky. For example, 32 percent of teachers report that they are "using games and other student-directed activities" more, though there is nothing in the standards suggesting such a shift. Conversely, 40 percent say they are using "flash cards and drills" *less*, though nothing in the standards explicitly discourages the use of these tools (especially since they are consistent with the expectation that students know their multiplication facts from memory by the end of grade 3). (For data on calculators, see *Calculator Use*.) These changes highlight the degree to which the standards themselves may be open to interpretation—and, in some cases, misinterpretation.

Table 14 • Teachers are using different strategies

Compared to before the CCSS-M were implemented, are you now doing more, about the same, or less of each of the following in your classroom?

	Total	K-2	3-5	6-8
Teaching using the number line				
More	37	36*	47*	27*
About the same	48	47	39	57*
Less	11	14	10	9
Using flash cards and drills**				
More	8	12	9	4*
About the same	34	31	41*	30
Less	40	45	39	36
Using games and other student-directed activities				
More	32	43*	30	25
About the same	44	38*	46	47
Less	20	17	20	23

* Indicates statistically significant difference at the 95 percent confidence level when comparing grade bands.
** Twenty-eight percent of 6–8 grade teachers say using flash cards and drills "doesn't happen."

Calculator Use

As Table 15 shows, elementary school teachers are seeing a modest shift away from calculator use, while middle school teachers are seeing an even bigger shift toward calculators. (Note, though, that large percentages of teachers in grades K–5 say they aren't sure about calculator use.) On the whole, these numbers are encouraging, since much of elementary school is focused on students learning to master operations, a time when calculators are not appropriate.

Table 15 • Teachers on calculators

Compared to before the CCSS-M were implemented, do you think you now have more, about the same number, or fewer of the following kinds of students in your classroom?

	Total	K-2	3-6	6-8
Students who rely on calculators				
More	20	3	8	44*
About the same	36	22*	40	44
Fewer	19	23	27	10*
Not sure	25	52*	25*	3*

* Indicates statistically significant difference at the 95 percent confidence level when comparing grade bands.

Teachers are more likely to report that the CCSS-M have changed *how* they teach than *what* they teach. Specifically, 52 percent say the standards have led to greater changes in "the way teachers teach" while 34 percent say they have led to greater changes in "the content teachers cover." Unfortunately, the overwhelming majority of teachers (85 percent) believes that these new approaches are hindering the reinforcement of learning at home because "parents don't understand the way math is being taught."

Standards Open to (Mis)Interpretation

"Common Core is not the answer. Students do not know the basics needed to function in their grade level. Multiplication tables need to be memorized." —**Grade 5, Massachusetts**

With the new standards, I no longer expect them to memorize formulas (they can look them up on Google!), but now expect them to know WHEN to use the formulas and WHY the formulas make sense. I truly love teaching math with these new standards!
—**Grade 6, Connecticut**

Bigger Changes at High-Poverty Schools

Teachers in high-poverty schools are particularly likely to report that the CCSS-M have changed "the way teachers teach." And they are more likely to report that their own teaching practice has changed. Specifically, compared to students in low-poverty schools, teachers in high-poverty schools are more likely to say they are doing more of the following as a result of the CCSS-M:

- Teaching multiple methods to solve a problem (62 percent vs. 41 percent)
- Requiring students to use proper math vocabulary (61 percent vs. 47 percent)
- Teaching using the number line (40 percent vs. 23 percent)
- Using games and other student-directed activities (38 percent vs. 21 percent)

Unhappy Parents

Parents at my school are very frustrated with math homework. We have had irate parents complaining about not understanding their child's math homework and refusing to help. We have abandoned sending math homework home for this reason. —**Grade 1, Delaware**

Increased Familiarity with the CCSS-M Is Associated with Greater Shifts in Practice

Teachers who have been teaching to the CCSS-M for four years are more likely than those who have taught to them for one year to say they are doing more of the following:

- Requiring students to explain in writing how they got their answers (67 percent vs. 53 percent)
- Requiring students to use proper math vocabulary (61 percent vs. 49 percent)
- Teaching multiple methods to solve a problem (57 percent vs. 48 percent)
- Teaching using the number line (41 percent vs. 30 percent)
- Using games and other student-directed activities (39 percent vs. 29 percent)

Question 3: What Impact Do Teachers Think the CCSS-M Are Having on Students' Mathematical Preparation?

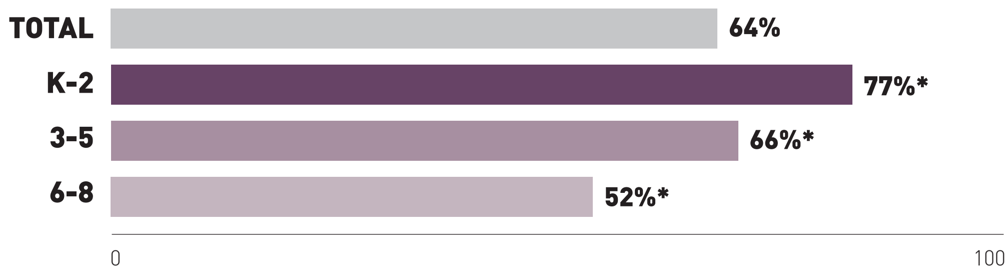
- *Teachers think students are developing better number sense.*
- *Teachers are divided over students' ability to perform "simple calculations."*
- *Teachers with students who are below grade level have a more negative view of the impact of the standards.*
- *Teachers think the new standards are stressful for students.*
- *A majority of teachers think the CCSS-M will have long-term benefits for students.*

Teachers think students are developing better number sense.

As shown in Table 16, 77 percent of K–2 teachers, and majorities of teachers in grades 3–5 and 6–8, agree that "students are developing a stronger number sense and more ability to apply math in real-world situations." This is a positive result that likely reflects the new standards' increased emphasis on conceptual understanding and application.

Table 16 • Teachers say students have better number sense

Students are developing a stronger number sense and more ability to apply math in real-world situations (Very or somewhat close to my view)



* Indicates statistically significant difference at the 95 percent confidence level when comparing grade bands.

Teachers are divided over students' ability to perform "simple calculations."

Consistent with the expectation that students be fluent in the standard algorithms for the four basic operations,[44] 32 percent of K-2 teachers say they have more students who can "do simple calculations with speed and accuracy" now than before the CCSS-M, while just 22 percent say they have fewer. However, this pattern is reversed in the higher grade bands, where larger numbers of teachers report that fewer students can do simple calculations (see Table 17). The results for middle school teachers are particularly concerning, with just 13 percent reporting that more students can perform simple calculations and 39 percent reporting that fewer can.

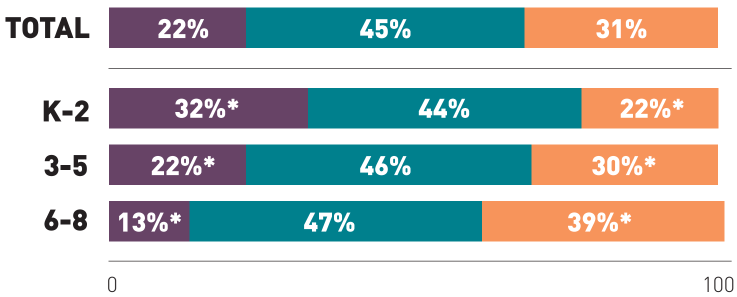
Table 17 • Teachers are divided over students' ability to do simple calculations

Q *Compared to before the CCSS-M were implemented, do you think you now have more, about the same number, or fewer of the following kinds of students in your classroom?*

More
About the same
Fewer

* Indicates statistically significant difference at the 95 percent confidence level when comparing grade bands.

Students who are able to do simple calculations with speed and accuracy



These results could mean that K–2 teachers are focusing on a smaller number of concepts *or* are misjudging the level of mastery that students are achieving. However, it's also possible that middle school teachers have a different understanding of "simple calculations" than teachers at lower levels or the results may be partly attributable to the higher expectations the standards place on students in these grades.

Teachers with students who are below grade level have a more negative view of the impact of the standards.

The 18 percent of survey respondents who primarily teach students who are remedial or significantly below grade level in math evince significantly more pessimism about the impact of CCSS-M on students than teachers who primarily teach on-grade-level students.

In particular, the former are:

- Less likely to agree that "students are developing a stronger number sense and more ability to apply math in real-world situations" (56 percent vs. 66 percent).
- More likely to report that fewer students "are able to do simple calculations with speed and accuracy" (44 percent vs. 28 percent).
- More likely to report that more students "consistently rely on the teacher for help" (51 percent vs. 38 percent).
- More likely to agree that "students are frustrated because they're being asked to learn many different ways to solve the same problem" (63 percent vs. 52 percent).

CCSS-M Stumbling Blocks

The most alarming aspect to the Common Core so far is the widening gap between the 'can do' kids and the 'can't do' kids. If a student struggles with reading and writing, it is an enormous stumbling block to the new math curriculum. —Grade 6, Florida

Although I value the concept behind teaching multiple ways to solve a problem, I am having a hard time teaching this way to my students who are far below proficiency. —Grade 4, Florida

Teachers think the new standards are stressful for students.

In general, teachers see the CCSS-M as a source of stress for students. As shown in Table 18, 42 percent of teachers overall say that they have more students with "math anxiety" than before the CCSS-M were implemented, and 53 percent agree that "expectations are unrealistic." In each of these cases, the higher the grade band, the more likely teachers are to report that students are encountering difficulties.

Table 18 • Teachers think the new standards are stressful for students

Q *Compared to before the CCSS-M were implemented, do you think you now have more, about the same number, or fewer of the following kinds of students in your classroom?*

	Total	K-2	3-5	6-8
Students who have math anxiety				
More	42	31*	45	48
About the same	39	39	38	41
Fewer	14	22*	13	8

Q *How close does the following statement come to your view on the broader impact of the CCSS-M beyond the classroom?*

	Total	K-2	3-5	6-8
Expectations are unrealistic; there will be too many students unable to reach these standards.				
Very or somewhat close to my view	53	41*	53*	61*

* Indicates statistically significant difference at the 95 percent confidence level when comparing grade bands.

Still, despite the stress they are creating, 58 percent of teachers overall believe the new standards benefit students in that "students are developing a stronger capacity to persevere in math and come up with solutions on their own" (Table 19). Middle school teachers in particular hold a more pessimistic view than K–5 teachers. (For more, see *What's Up with Middle School Teachers?*)

Table 19 • Students are learning to persevere in math

Q *How close does each of the following statements come to your view on the impact of the CCSS-M in the classroom?*

	Total	K-2	3-5	6-8
Students are developing a stronger capacity to persevere in math and come up with solutions on their own				
Very or somewhat close to my view	58	68	61	46*

* Indicates statistically significant difference at the 95 percent confidence level when comparing grade bands.

What's Up with Middle School Teachers?

Middle school teachers tend to have more of a negative assessment of students' math abilities. But why?

The most likely explanation is that the middle school standards are simply harder than the elementary school standards (especially relative to the standards they replaced). The CCSS-M place dramatically higher expectations on students once they enter middle school, and much of the content that is introduced in these grades is new for both teachers and students. Perhaps middle school teachers haven't received the professional development required to successfully navigate this transition. Or perhaps middle school students who have had to transition into the CCSS-M during elementary school failed to acquire the skills necessary to succeed in middle school math.

Interestingly, despite their generally negative assessment of students' abilities, many middle school teachers remain optimistic about the future of CCSS-M to prepare students for the future. Roughly half believe that students are getting better preparation for the advanced math needed to succeed in selective colleges and STEM majors (52 percent), and that the standards will help ensure that America's young people have the math skills needed to compete in a global economy (50 percent).

Change and Duress

Common Core is causing even students in honors classes to struggle because they continue to lack basic math skills needed for higher level thinking and computation.
—Grade 7, California

All of the complaints are because the expectations of the standards have exposed students, parents, and especially math teachers as lacking a fundamental understanding of what math is and how it should be taught for mastery and application. **—Grade 6, Florida**

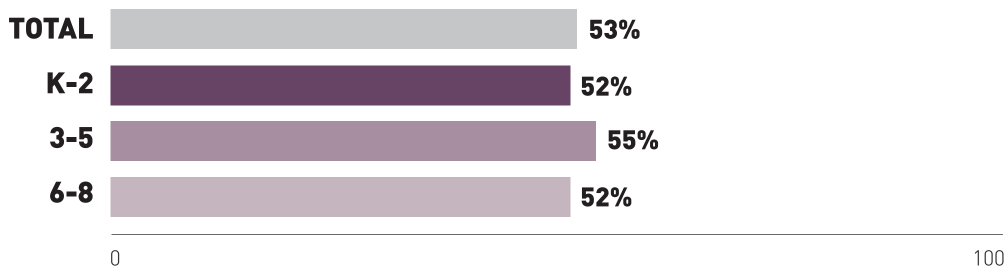
A majority of teachers think the CCSS-M will have long-term benefits for students.

As shown in Table 20, a majority of teachers (53 percent) report that the statement, "Students are getting better prepared for the advanced math needed to succeed in selective colleges or as STEM majors," is very or somewhat close to their view; while 34 percent say it is not close, and 14 percent are not sure. Similarly, a majority of teachers (55 percent) report that the statement, "the standards will help ensure that America's young people have the math skills needed to compete in the global economy," is very or somewhat close to their view; while 36 percent say that it is not close, and 9 percent are not sure.

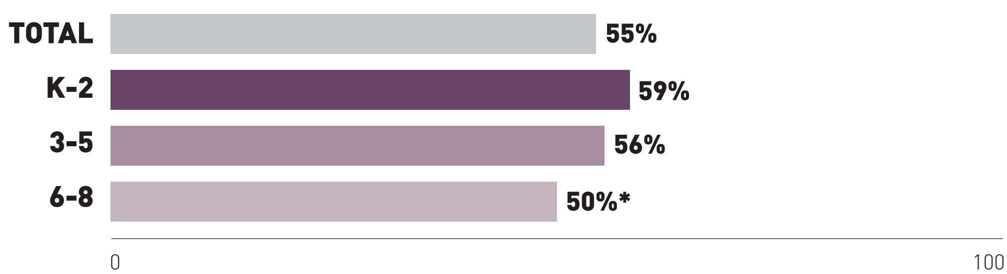
Table 20 • A majority of teachers think the CCSS-M will have long-term benefits for students

Q *How close does each of the following statements come to your view on the broader impact of the CCSS-M beyond the classroom? (Very or somewhat close to my view)*

Students are getting better prepared for the advanced math needed to succeed in selective colleges or as STEM majors



The standards will help ensure that America's young people have the math skills needed to compete in the global economy



* Indicates statistically significant difference at the 95 percent confidence level when comparing grade bands.

Surpassing Expectations

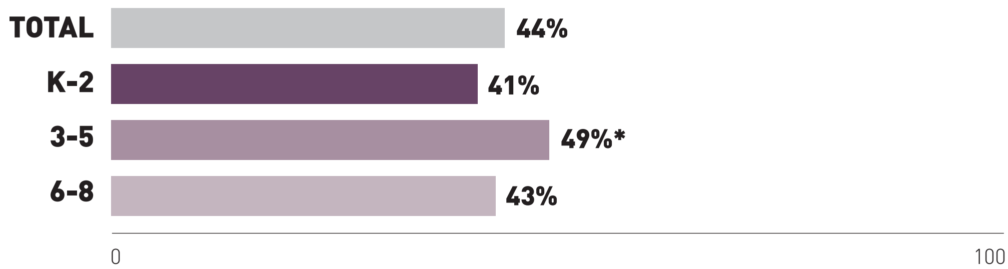
I didn't think second-graders were capable of learning everything that is required from the CCSS, but they are able to do it, and I feel they will be better prepared for the global economy. —Grade 2, Wyoming

Teachers are divided when it comes to how the CCSS-M may affect them as professionals. For example, although 44 percent agree that the standards "take away from the creativity and joy of teaching," 53 percent say this is not close to their view (Table 21). But a solid majority (62 percent) do agree that "teachers' math content knowledge will improve" as a result of the CCSS-M, while 32 percent say this does not come close to their view.

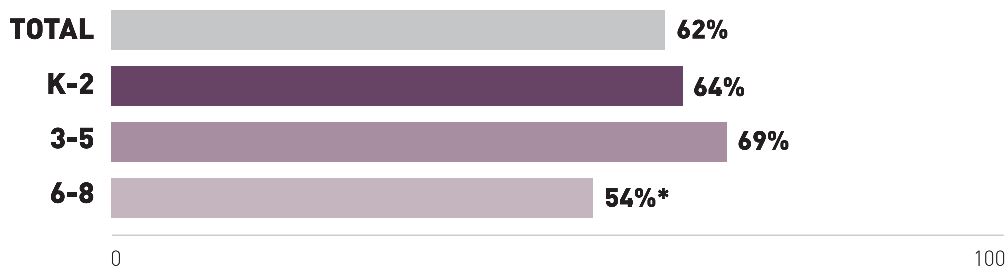
Table 21 • Teachers on how CCSS-M impacts them

Q *How close does each of the following statements come to your view on the impact of the CCSS-M in the classroom/beyond the classroom? (Very or somewhat close)*

The standards take away from the creativity and joy of teaching



Teachers' math content knowledge will improve



* Indicates statistically significant difference at the 95 percent confidence level when comparing grade bands.

Growing Pains

Right now, we are in the midst of some serious growing pains. Also, many math teachers will have to improve their own understanding of mathematics in order to adequately teach their students. —GRADE 7, FLORIDA

Discussion and Takeaways

The CCSS-M are designed to support a more focused, coherent, and rigorous instructional program that places greater emphasis on conceptual understanding and real-world application, in addition to procedural fluency. But are these shifts occurring at the classroom level? Overall, the survey results suggest they may be, though there is still much work to be done and much about how the standards are being implemented that we don't yet understand. Below we consider what the results imply for each of the three CCSS-M shifts: focus, coherence, and rigor.

Focus

A primary goal of the standards is replacing the "mile wide and inch deep" mentality of yesteryear with an intense focus on the topics most essential for college and career readiness. Achieving this goal requires that teachers do at least two things effectively: cover the appropriate grade-level topics and devote the most time and attention to those considered most important.

On both of these counts, our results suggest that most teachers are succeeding. For example, across grades, thirty-seven of the forty-four major grade-level clusters included in the survey were identified by at least 90 percent of teachers from the appropriate grade as among those they would teach by the end of the school year. And in all but one grade, four of the five clusters that teachers were most likely to say they would focus on were major grade-level clusters.[45] Together, these results suggest that teachers, on the whole, are focusing on the most important grade-level topics, as described in the standards.

Encouragingly, two-thirds of teachers do *not* agree that "important math topics are being missed as a result of a narrower curriculum." Yet some teachers express concern about the length of time required to teach multiple approaches to the same problem, suggesting that actually achieving a focused curriculum remains a challenge.

Slow Pressure

> *Teaching all the strategies makes for very, very slow teaching and with the pressures of getting through a curriculum it is hard to make it work. I didn't get through my entire curriculum last year also. A lot of other teachers in our school didn't because there's just no time with the way we teach math now.* —**Grades 2 and 3, Maryland**

Coherence

Another goal of the CCSS-M is achieving greater coherence across grades by connecting the topics and concepts covered in one grade to those in the next (while avoiding unnecessary repetition). Yet achieving this degree of coordination across grades is tricky, as is assessing the progress that has been made on this front.

On the one hand, the fact that teachers are generally covering the appropriate grade-level topics has positive implications for coherence (as it does for focus). Similarly, it's encouraging that a majority of teachers say their math curriculum is consistent for all math classes in their grade. After all, if teachers in the same grade are using different curricula, or covering different topics, the task of "linking" becomes more complicated for their colleagues in the grades above.

On the other hand, because the term "curriculum" is open to interpretation, it's difficult to know what teachers mean when they say their curricula are or aren't the same, since they could be referring to the topics they cover, the textbooks they use, district- or school-provided resources, or something in between. Additionally, because linking concepts may require that teachers review material from previous grades, evidence that teachers are teaching topics that aren't on grade level must be interpreted cautiously, since linking is difficult to distinguish from "repeating." Most concerning is the lack of attention to working with colleagues at different grades. Knowing what has been taught (and how it was taught) requires more than a review of the standards or district curriculum; it requires collaboration with teachers in neighboring grades.

When asked directly, 38 percent of teachers report doing more "linking new math concepts to those taught in earlier grades" than before the CCSS-M were implemented, while just 5 percent say they do less of this—an encouraging ratio. Still, given the qualifiers outlined above, it's difficult to know if the level of coherence has actually increased. At the very least, the results don't raise red flags.

Rigor

Of the three shifts required by the CCSS-M, promoting rigor by pursuing conceptual understanding, procedural skill and fluency, and application with "equal intensity" may be the most challenging to implement—and the survey results reflect this tension.

On the one hand, there is strong evidence that teachers are attending more to applications of mathematics—a positive finding inasmuch as students need to be able to use what they know. Similarly, teachers seem to be attending to mathematical concepts more than they used to, often in ways that are likely to benefit students. For example, having students explain their thought process and use correct mathematical terminology can solidify important concepts. And talking with students about more than one approach to a problem can reinforce learning (especially if the emphasis is on how the methods relate to one another mathematically, rather than committing still more procedures to memory).

On the other hand, the survey responses also suggest that teachers may be diluting students' mastery by covering too many strategies. Knowing alternate strategies requires conceptual understanding and strengthens procedural fluency. When students know several ways to solve a problem, they are able to select the most appropriate and efficient strategy given the values that appear in it. Yet teachers nonetheless report frustration on the part of students who are taught multiple methods. As this discussion highlights, achieving rigor in the classroom means finding a useful balance between conceptual understanding, procedural fluency, and application—something the results suggest teachers are still grappling with.

Implications

The survey results and discussion above imply at least five takeaways for both teachers and other local and state education officials.

1. Stay the course—change takes time.

The changes that need to take place to implement CCSS-M with fidelity won't happen overnight. It will take time for teachers to understand and prioritize the new grade-level topics and fully familiarize themselves with the Standards for Mathematical Practice. Getting on the same page with grade-level colleagues—much less teachers in other grades and subjects—is a learned habit. And it can take years for teachers to get accustomed to the ins and outs of new curricula and instructional materials, which may also be changing.

The Real Foe

> *Common Core is not the enemy. The implementation and follow through is the problem.*
> **—Grade 7, Utah**

Overall, the results suggest the transition to CCSS-M has been hardest at the middle school level, where students actually did have to *transition*. Elementary teachers at the K–2 level (especially those working with children who have known only the CCSS-M) are more likely to describe positive outcomes. And teachers who have taught to the CCSS-M for longer are more likely to report having adjusted their pedagogy. These results suggest that both students and teachers will ultimately benefit from staying the course.

2. Increase the amount of time devoted to collaboration across grade levels.

Teachers are generally more likely to say that they are doing more linking math concepts across grades than before the CCSS-M—but it's hard to know whether these links are functioning as a bridge to grade-level content or if they are a repetition of what should have already been covered. Obviously, teachers who work closely together would know which of the two is occurring; yet the survey results suggest that collaboration among teachers in different grade levels has not been a priority during the implementation of the CCSS-M.

According to the CCSS-M architects, "the development of the standards began with research-based learning progressions detailing what is known today about how students' mathematical knowledge, skill, and understanding develop over time."[46] In other words, the design of the standards *assumes* that content taught in one grade will connect to content taught in the next, because this is how mathematics is learned. Given this central assumption of the standards, it seems even more essential that teachers across grade levels be granted structured opportunities to align their teaching with one another.

3. Keep parents informed, and make homework comprehensible.

It is no secret that the Common Core has generated pushback in some parts of the country, in part because of the perception that it represents a radical departure from the way math has traditionally been taught. A whopping 85 percent of teachers say that the statement "Reinforcement of math learning at home is declining because parents don't understand the way math is being taught" is very or somewhat close to their view. Regardless of whether this confusion is warranted, this perception must change if the standards are to have the desired effect.

On this front, two steps would make a world of difference: First, teachers and principals should keep parents informed about the purpose of the standards and engaged in their children's math education. As Jason Zimba, lead writer of the CCSS-M, has suggested, there are many ways in which parents can be involved, even if math isn't their strong suit, including helping with skill building and fluency practice.[47]

Second, make homework assignments as straightforward and comprehensible as possible, so that parents can understand them. More important than *teaching a method* and *practicing a method* (especially one not familiar to families), is ensuring that a student selects *the method* that makes sense to her and from which she can efficiently and accurately reach a solution. Parents are then able to support the student with methods they know. If the goal of the homework is to provide practice with a *new method*, then teachers should support families by sending home worked examples of new methods or providing online links that explain the method, among other strategies.

4. Press curriculum developers not only for better-aligned materials but for those that help educators teach students who are below grade level.

Though the survey results suggest that most teachers now have access to materials that are aligned to the CCSS-M, we have a long way to go. And mere alignment is not enough. Teachers also need materials that can help them reach students who are below grade level, who may be even further behind than they were before the CCSS-M now that the bar has been raised. (It is likely no coincidence that teachers who teach below-grade-level students have a more negative view of the standards.) Educators have the contradictory task of getting these students up to speed (which requires moving quickly through the content) while teaching with increased rigor (which compels them to slow down). They will need help to square this circle.

The "K–8 Publishers' Criteria for the Common Core State Standards for Mathematics"[48] envisions curricular materials that "manage unfinished learning from earlier grades inside grade-level work, rather than setting aside grade-level work to re-teach earlier content." Further:

> *[U]nfinished learning from earlier grades is normal and prevalent; it should not be ignored nor used as an excuse for cancelling grade-level work and retreating to below-grade work. For example, the development of fluency with division using the standard algorithm in grade 6 is the occasion to surface and deal with unfinished learning about place value; this is more productive than setting aside division and backing up.*

A curriculum that manages to accommodate struggling students without sacrificing on-grade-level work seems too good to be true, but it shouldn't be. We should evaluate materials not only on their alignment to the CCSS-M, but on their approach to and success in bringing below-grade-level students up to speed.[49]

5. When the CCSS-M are revised, work to address some of the problems identified here.

Our results reveal several challenges associated with implementing the CCSS-M. First, teachers are struggling to effectively pursue rigor. It is no easy task to balance the treatment of conceptual understanding, procedural skill and fluency, and application. Second, some teachers still think that the standards include more content than they have time to teach well. Third, achieving fluency requires a deep understanding of the content to be taught in other grades; the lack of cross-grade collaboration makes it harder for teachers to attain that. Fourth, though the CCSS-M specifies that young students will know from memory their addition and multiplication facts, it is not clear whether teachers understand the role memorization might play. We need to clarify the goals for basic fact instruction so there are clear connections between using mental strategies and reaching mastery. Finally, though they are generally on board with teaching students multiple methods to solve a problem, some teachers nonetheless think it confuses students rather than deepens their understanding.

There are no easy answers when it comes to addressing these challenges, some of which are likely best handled if and when the standards are revised. Given that this revision likely won't occur any time soon, curriculum designers and professional development staff have their work cut out for them in the interim.[50]

Final Thoughts

For the first time in our nation's history, there is a high level of consistency regarding what's taught in American elementary and middle school math classrooms. Fewer teachers appear to be closing their classroom doors and doing their own thing.

Most teachers think the new standards are more rigorous than their old ones, are teaching the major CCSS-M topics at their grade level, and are incorporating different ways of teaching mathematics. Across grades, there is support for the vision of the Common Core State Standards, though not without concerns about student preparedness, especially at higher grade levels.

Meanwhile, students are being exposed to fewer topics in more depth, spending significant time on applications in mathematics, and learning in different ways. Much of that time is being spent on number and algebra-related content, as reflected in the design of the standards.

Yet eluding many teachers is the just-right balance between conceptual understanding, fluency of procedures, and the application of mathematics. Getting this balance right is key not only to ending the math wars but to developing mathematically proficient, globally competitive, college- and career-ready students—in other words, ensuring that the Common Core is properly implemented and accomplishes its intended purpose.

Appendix A: Methodology

This report is based on an online survey of a representative sample of 1,003 K–8 public school teachers who teach math. It includes teachers from the forty-three states (plus the District of Columbia) that had adopted the Common Core State Standards for Mathematics (CCSS-M) as of March 2015. The excluded non-adopting states are Alaska, Indiana, Minnesota, Nebraska, Oklahoma, Texas, and Virginia.[51]

The survey was fielded online between March 30 and May 15, 2015. The margin of error for the overall sample is plus or minus 3 percentage points; it is plus or minus 6 percentage points for each grade band (K–2, 3–5, and 6–8) and plus or minus 10 percentage points for each individual grade (Kindergarten through grade 8). The survey was preceded by three focus groups (more below). Both the survey and focus groups were conducted by the Farkas Duffett Research Group (FDR Group) on behalf of the Thomas B. Fordham Institute.

Teachers' names were randomly drawn from a comprehensive national database of K–12 educators maintained by Agile Education Marketing located in Colorado. As indicated, records from Alaska, Indiana, Minnesota, Nebraska, Oklahoma, Texas, and Virginia were excluded. Geographically representative samples of public school teachers from the remaining records were drawn based on grade level for Kindergarten through grade 5 teachers, and based on both grade level and subject (math) for 6–8 grade teachers.

The survey instrument was designed, pre-tested, and prepared for administration online. Potential respondents were sent invitations via email; a unique link to the survey was embedded in each invitation. The original message was sent on March 30 and 31, 2015, to all potential respondents. Between April 4 and May 7, seven reminder messages were sent to different waves of non-respondents. In addition, 105 K–5 teachers were randomly selected and received a telephone call or message that encouraged them to keep an eye out for the email invitation and to complete the survey.

This approach yielded 1,003 completed interviews. Final sample dispositions are reported in Table A-1. In the end, the survey garnered a cooperation rate of 39 percent (1,003 complete + 698 partial + 63 not qualified) / (1,003 complete + 698 partial + 63 not qualified) + (2,511 opened/no click + 304 opted-out). That is to say, of those teachers who opened the email invitation, approximately two out of five attempted or completed the survey.[52]

Table A-1 • Final sample disposition

Messages Attempted	42,719
Complete	1,003
Partial	698
Not qualified	63
Opened (no click)	2,511
Bounced-back	2,412
Opted-out	304
Unknown	35,728

The survey results are weighted to reflect the actual distribution of teachers across regions (Northeast, Midwest, South, West). Weighting compensates for factors in the survey design and implementation that would otherwise skew or bias estimates drawn from the survey data.

Table A-2 compares the demographic profiles of weighted and unweighted samples.

Table A-2: Demographics of weighted and unweighted samples

	Weighted %	*Unweighted %*
Urbanicity		
Urban	26	28
Suburban	39	37
Rural	35	35
Region		
Northeast	25	13
Midwest	22	14
South	31	42
West	22	32
Percentage of Students Eligible for Free or Reduced-Price Lunch		
0–24%	18	15
25–49%	24	23
50–74%	26	27
75–100%	32	36
Sex		
Male	13	12
Female	88	88

As with all surveys, the risk of non-response is that the pool of survey respondents could differ from the true population of teachers, decreasing the ability to draw inferences from the data. Results can also be affected by non-sampling sources of bias, such as question wording and order. Steps were taken to minimize these threats, including extensive pre-testing of the survey instrument and randomization of survey items and answer categories.

The survey was programmed and the data collected by JAIM Research Services of New Jersey; the data were tabulated by Clark Research of South Dakota. The questionnaire was crafted by the FDR Group in conjunction with the Thomas B. Fordham Institute.

Focus Groups

To help develop the questionnaire, three focus groups were conducted in Fall 2014: one with K–2 grade teachers in Baltimore, MD; one with 3–5 grade teachers in Walnut Creek, CA; and one with 6–8 grade teachers in Louisville, KY. The purpose of the focus groups was to gain firsthand understanding of the views of math teachers, to develop new hypotheses based on their input, and to design the survey items using language and terms for which teachers are comfortable. Participants were recruited to ensure an appropriate demographic mix of teachers by grade, socio-economic status of schools, and urbanicity. All focus groups were moderated by the FDR Group.

Appendix B: Complete Survey Results

K–8 Public School Math Teachers

The online survey was fielded from March 30, 2015 – May 15, 2015 and includes a total of 1,003 respondents. Note that numbers may not total to 100 percent due to rounding. An asterisk indicates less than 0.5 percent, and a dash indicates zero.

Q1. Do you teach all or most subjects, or do you specialize in any subject below?

54 All or most subjects
47 Math

Q2. Which grades do you teach this school year (2014–2015)?

10 Kindergarten
10 Grade 1
11 Grade 2
11 Grade 3
12 Grade 4
10 Grade 5
11 Grade 6
12 Grade 7
12 Grade 8

Base: Grades K–5 (n=649)

Q3. Would you say MOST of your students are:

76 On or close to grade level
3 Gifted or significantly above grade level
21 Remedial or significantly below grade level

Base: Grades 6–8 (n=354)

Q4. Do you teach any regular or on-grade-level math classes this school year, or not?

84 Yes
16 No

Base: 6–8 grade teachers who do not teach any regular or on-grade-level math classes (n=62) (Does not total to 100 percent due to multiple responses)

Q5. Do you teach any math classes this school year where most students are:

48 Gifted or significantly above grade level
57 Remedial or significantly below grade level
20 Special education

Q6. What region is your school in?

25 Northeast
22 Midwest
31 South
22 West

Q7. As far as you know, what year were the CCSS-M for your grade implemented at your school?

16 2014–2015
38 2013–2014
30 2012–2013
15 2011–2012
2 Not yet implemented

Base: CCSS-M implemented in 2014–2015, 2013–2014, 2012–2013, and 2011–2012 (n=984)

Q8. Would you say that the CCSS-M for your grade at your school are:

66 Fully implemented
26 Mostly implemented
7 Partially implemented
* Barely implemented

Q9. There have been a variety of math standards adopted by states across the country. During your classroom teaching career, have you also taught math based on other standards, or have you always taught math based on the CCSS-M?

85 Also based on other standards
15 Always based on the CCSS-M

Q10. Please select the primary math curriculum used in your classroom this school year.

Base: Grade=K, 1, 2 (n=318)

10 Engage New York/The New York State Education Department
16 Envision Math/Scott Foresman-Pearson
1 Eureka Math/Great Minds
11 Everyday Mathematics/Everyday Learning-McGraw-Hill
15 Go Math/Houghton Mifflin Harcourt
2 Harcourt Math or HSP Math/Houghton Mifflin Harcourt
6 Investigations in Number, Data, and Space/Scott Foresman-Pearson
1 Journeys/Houghton Mifflin Harcourt
2 Math Connects/Glencoe-McGraw-Hill
6 Math Expressions/Houghton Mifflin Harcourt
4 Math in Focus or Singapore Math/Houghton Mifflin Harcourt
- Math Trailblazers/Kendall Hunt
3 My Math/McGraw-Hill
1 Saxon Math/Saxon-Houghton Mifflin Harcourt
7 Self-developed curriculum that pulls from multiple sources
3 School-developed curriculum that pulls from multiple sources
5 District- or County-developed curriculum that pulls from multiple sources
2 State-developed curriculum that pulls from multiple sources
7 Something else (volunteered responses: Origo, AMSTI, Bridges)

Base: Grade=3, 4, 5 (n=331)

10 Engage New York/The New York State Education Department
15 Envision Math/Scott Foresman-Pearson
1 Eureka Math/Great Minds
5 Everyday Mathematics/Everyday Learning-McGraw-Hill
16 Go Math/Houghton Mifflin Harcourt
- Growing with Math/Glencoe-McGraw-Hill
1 Harcourt Math or HSP Math/Houghton Mifflin Harcourt
3 Houghton Mifflin Mathematics/Houghton Mifflin Harcourt
8 Investigations in Number, Data, and Space/Scott Foresman-Pearson
- Journeys/Houghton Mifflin Harcourt
* Math Connects/Glencoe-McGraw-Hill
5 Math Expressions/Houghton Mifflin Harcourt
5 Math in Focus or Singapore Math/Houghton Mifflin Harcourt
1 Math Trailblazers/Kendall Hunt
4 My Math/McGraw-Hill
2 Saxon Math/Saxon-Houghton Mifflin Harcourt
2 Scott Foresman-Addison Wesley Mathematics/Scott Foresman-Pearson
6 Self-developed curriculum that pulls from multiple sources
1 School-developed curriculum that pulls from multiple sources
8 District- or County-developed curriculum that pulls from multiple sources
2 State-developed curriculum that pulls from multiple sources
5 Something else (volunteered responses: Origo, Bridges, Georgia Math)

Base: Grade=6, 7, 8 (n=354)

1 Agile Mind/Common Core Middle School Mathematics
1 Algebra I/McDougal Littell-HMH (Larson)
1 Algebra I/Prentice Hall-Pearson

9 Big Ideas Learning/Big Ideas Math
1 Bridge to Algebra or Carnegie Learning/Carnegie Learning
* College Preparatory Math/CPM
2 College Preparatory Math/CPM Core Connections
2 Connected Math/Pearson
4 Connected Math/Prentice-Hall-Pearson
5 Digits/Pearson
- Edgenuity/Edgenuity Inc.
5 Engage New York/The New York State Education Department
- Envision Math/Scott Foresman-Pearson
1 Eureka Math/Great Minds
5 Go Math/Houghton Mifflin Harcourt
8 Holt McDougal Math/Houghton Mifflin Harcourt
- Journeys/Houghton Mifflin Harcourt
11 Math Connects/Glencoe-McGraw-Hill
1 Math in Focus or Singapore Math/Houghton Mifflin Harcourt
2 Prentice Hall Math/Prentice Hall-Pearson
2 Springboard Mathematics/College Board
10 Self-developed curriculum that pulls from multiple sources
5 School-developed curriculum that pulls from multiple sources
8 District- or County-developed curriculum that pulls from multiple sources
2 State-developed curriculum that pulls from multiple sources
14 Something else (volunteered responses: Core Focus on Math, Scholastic, Georgia Math)

Q11–16. Thinking about the math curriculum used in your classroom this school year:

% responding	Yes	No	Not Sure
[Q11] Is it the same for all math classes in your grade, or not?	82	17	1
[Q12] Is it the same for all grades throughout your school, or not?	61	31	8
[Q13] Do you have a primary textbook, or not?	62	37	1
[Q14] Are you required to use the textbook, or not? (Base: Has textbook n=619)	51	46	3
[Q15] Are you provided with instructional materials, or not? (e.g., guides, workbooks, manipulatives)	84	16	1
[Q16] Are you required to use the instructional materials, or not? (Base: Provided instructional materials n=839)	49	47	4

Q17. Generally speaking, when it comes to deciding which math topics to cover (content), do you:

44 Tend to follow the curriculum and instructional materials as is
55 Often modify the curriculum and instructional materials
1 Not sure

Base: "Often Modify" (n=560)

Q18. And when you modify the curriculum and instructional materials in deciding which math topics to cover, is it mainly:

81 Because the learning needs of your students demand it
5 Because the math department or district tells you to
14 Something else (volunteered responses: both, to align to CCSS-M and state tests)
* Not sure

Q19. Generally speaking, when it comes to deciding the amount of time to allot to each math topic (pacing), do you:

32 Tend to follow the curriculum and instructional materials as is
67 Often modify the curriculum and instructional materials
1 Not sure
Base: "Often Modify" (n=676)

Q20. And when you modify the curriculum and instructional materials in deciding the amount of time to allot to each math topic, is it mainly:

92 Because the learning needs of your students demand it
4 Because the math department or district tells you to
4 Something else (volunteered responses: both, time constraints)
- Not sure

Q21. Generally speaking, when it comes to deciding the order of math topics (sequence), do you:

64 Tend to follow the curriculum and instructional materials as is
35 Often modify the curriculum and instructional materials
1 Not sure
Base: "Often Modify" (n=349)

Q22. And when you modify the curriculum and instructional materials in deciding the order of math topics, is it mainly:

62 Because the learning needs of your students demand it
16 Because the math department or district tells you to
22 Something else (volunteered responses: both, CCSS-M/state tests, pacing guides, to approach material more logically)
* Not sure

Q23–27. Compared to before the CCSS-M were implemented, are you now doing more, less, or about the same amount of discussing math curriculum and instructional practices with the following teachers?

% responding	Base	More	Less	About Same	Doesn't Happen	Not Sure
[23] With teachers in your grade	K–5 (649)	52	5	40	2	2
[24] With teachers in other grades	K–5 (649)	28	5	44	20	4
[25] With math teachers in your grade	6–8 (354)	57	4	34	3	2
[26] With math teachers in other grades	6–8 (354)	37	8	45	8	2
[27] With teachers of other subjects in your grade	6–8 (354)	18	13	49	16	4

Q28–30. How would you rate the teachers at your school when it comes to knowing what the focus topics are for the CCSS-M in the following grades?

% responding	Excellent	Good	Only Fair	Poor	Not Sure
[Q28] In their own grade	47	43	7	1	2
[Q29] In the grade immediately below their own	10	43	29	10	8
[Q30] In the grade immediately above their own	11	42	28	10	9

Q31–40. Compared to before the CCSS-M were implemented, are you now doing more, less, or about the same of each of the following in your classroom?

% responding	More	Less	About Same	Doesn't Happen	Not Sure
[Q31] Integrating math concepts into other subjects you teach	23	7	51	14	5
[Q32] Linking new math concepts to those taught in earlier grades	38	5	51	2	4
[Q33] Prioritizing computation	23	21	48	3	5
[Q34] Requiring students to explain in writing how they got their answers	64	3	29	3	1
[Q35] Requiring students to use proper math vocabulary	55	2	43	*	1
[Q36] Teaching multiple methods to solve a problem	56	3	39	1	1
[Q37] Teaching multi-step word problems	58	3	36	1	2
[Q38] Teaching using the number line	37	11	48	4	1
[Q39] Using flash cards and drills	8	40	34	17	1
[Q40] Using games and other student-directed activities	32	20	44	3	1

Q41–50. Compared to before the CCSS-M were implemented, do you think you now have more, fewer, or about the same number of the following kinds of students in your classroom?

% responding	More	Fewer	About Same	Not Sure
[Q41] Students who have math anxiety	42	14	39	5
[Q42] Students who are able to do simple calculations with speed and accuracy	22	31	45	2
[Q43] Students who can apply the correct math procedures in word problems without prompting from the teacher	24	27	47	3
[Q44] Students who have enthusiasm for learning math	20	28	50	2
[Q45] Students who memorize basic math formulas or times tables	9	40	45	7
[Q46] Students who rely on calculators	20	19	36	25
[Q47] Students who consistently rely on the teacher for help	40	14	44	2
[Q48] Students who can solve challenging problems that require addition and subtraction and an understanding of place value (K–2, n=315)	37	14	43	7
[Q49] Students who can solve challenging problems that require multiplication and division with whole numbers and fractions (3–5, n=325)	28	24	46	2
[Q50] Students who understand and can solve challenging problems related to ratios and proportions (6–8, n=353)	27	27	45	2

Q51. Here is a list of math topics that may or may not be covered in your grade. Which of these math topics will you teach by the end of this school year? (Data for Q51 and Q52 have been combined in the following tables.)

Q52. Which of these math topics will you spend the MOST amount of time on during this school year? Please select up to five.

Kindergarten (n=103) *Level: major, supporting, additional*	**Q51** **Will Teach**	**Q52** **Most Time**	**Grade Level (type of cluster)**
Classify objects and count the number of objects in categories	95	20	K (supporting)
Compare numbers	98	46	K (major)
Connect counting to cardinality	94	44	PK
Count numbers to 20 by rote	94	22	PK
Count to tell the number of objects	95	52	K (major)
Describe and compare measureable attributes	91	5	K (additional)
Identify and describe shapes	95	20	K (additional)
Identify positions of objects and people in space, including in/on/ under, up/down, inside/outside, beside/between, in front/ behind	92	3	PK
Know number names and the count sequence	95	44	K (major)
Measure and estimate lengths in standard units	67	5	2
Represent and solve problems involving addition and subtraction	97	62	1
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from	95	58	K (major)
Work with numbers 11–19 to gain foundations for place value	92	53	K (major)
Work with time and money	26	4	2
Something else	13	5	
None of these	-		
Not sure		-	

Grade 1 (n=104) *Level: major, supporting, additional*	**Q51** **Will Teach**	**Q52** **Most Time**	**Grade Level (type of cluster)**
Add and subtract within 20	96	86	1 (major)
Count numbers to 20 by rote	76	7	PK
Count to tell the number of objects	77	12	K
Develop understanding of fractions as numbers	50	3	3
Draw and identify lines and angles, and classify shapes by properties of their lines and angles	35	2	4
Extending the counting sequence	83	45	1 (major)
Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures	14	3	3, a
Identify and describe shapes	89	1	K
Measure lengths indirectly and by iterating length units	77	9	1 (major)
Reason with shapes and their attributes	75	6	1 (additional)
Represent and solve problems involving addition and subtraction	92	84	1 (major)
Represent and solve problems involving multiplication and division	13	3	3
Tell and write time	90	13	1 (additional)

Use place value understanding and properties of operations to add and subtract	90	79	1 (major)
Something else	13	2	
None of these	-		
Not sure		1	

Grade 2 (n=108) *Level: major, supporting, additional*	Q51 Will Teach	Q52 Most Time	Grade Level (type of cluster)
Add and subtract within 20	95	63	2 (major)
Classify objects and count the number of objects in categories	53	2	K
Develop understanding of fractions as numbers	60	4	3
Extending the counting sequence	80	23	1
Measure and estimate lengths in standard units	90	12	2 (major)
Measure lengths indirectly and by iterating length units	60	1	1
Reason with shapes and their attributes	87	5	2 (additional)
Represent and solve problems involving addition and subtraction	95	75	2 (major)
Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit	45	4	4
Use place value understanding and properties of operations to add and subtract	98	85	2 (major)
Use the four operations with whole numbers to solve problems	27	10	4
Work with equal groups of objects to gain foundations for multiplication	87	14	2 (supporting)
Work with time and money	94	41	2 (supporting)
Something else	10	1	
None of these	-		
Not sure		1	

Grade 3 (n=108) *Level: major, supporting, additional*	Q51 Will Teach	Q52 Most Time	Grade Level (type of cluster)
Develop understanding of fractions as numbers	94	51	3 (major)
Extend understanding of fraction equivalence and ordering	91	25	4
Geometric measurement: understand concepts of area and relate area to multiplication and to addition	95	23	3 (major)
Graph points on the coordinate plane to solve real-world and mathematical problems	32	-	5
Measure and estimate lengths in standard units	86	1	2
Reason with shapes and their attributes	85	3	3 (supporting)
Represent and solve problems involving multiplication and division	95	76	3 (major)
Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects	81	9	3 (major)
Solve problems involving the four operations, and identify and explain patterns in arithmetic	90	68	3 (major)
Understand decimal notation for fractions, and compare decimal fractions	11	5	4
Understand properties of multiplication and the relationship between multiplication and division	96	78	3 (major)
Use place value understanding and properties of operations to add and subtract	91	41	2

Use place value understanding and properties of operations to perform multi-digit arithmetic	83	37	3 (additional)
Work with time and money	77	10	2
Something else	11	3	
None of these	-		
Not sure		1	

Grade 4 (n=116) *Level: major, supporting, additional*	Q51 Will Teach	Q52 Most Time	Grade Level (type of cluster)
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers	93	53	4 (major)
Develop understanding of fractions as numbers	95	60	3
Draw and identify lines and angles, and classify shapes by properties of their lines and angles	95	11	4 (additional)
Extend understanding of fraction equivalence and ordering	96	56	4 (major)
Gain familiarity with factors and multiples	96	36	4 (supporting)
Generalize place value understanding for multi-digit whole numbers	97	42	4 (major)
Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition	52	2	5
Graph points on the coordinate plane to solve real-world and mathematical problems	40	-	5
Measure and estimate lengths in standard units	88	2	2
Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit	87	11	3
Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects	70	6	4 (supporting)
Understand decimal notation for fractions, and compare decimal fractions	93	28	4 (major)
Understand ratio concepts and use ratio reasoning to solve problems	17	1	6
Use the four operations with whole numbers to solve problems	99	74	4 (major)
Something else	6	3	
None of these	-		
Not sure		5	

Grade 5 (n=100) *Level: major, supporting, additional*	Q51 Will Teach	Q52 Most Time	Grade Level (type of cluster)
Apply and extend previous understandings of multiplication and division to multiply and divide fractions	92	62	5 (major)
Classify two-dimensional figures into categories based on their properties	88	2	5 (additional)
Convert like measurement units within a given measurement system	84	14	5 (supporting)
Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition	93	12	5 (major)
Graph points on the coordinate plane to solve real-world and mathematical problems	97	11	5 (additional)
Investigate chance processes and develop, use, and evaluate probability models	35	2	7
Perform operations with multi-digit whole numbers and with decimals to hundredths	99	54	5 (major)

Represent and analyze quantitative relationships between dependent and independent variables	32	1	6
Understand decimal notation for fractions, and compare decimal fractions	91	53	4
Understand properties of multiplication and the relationship between multiplication and division	97	51	3
Understand ratio concepts and use ratio reasoning to solve problems	33	3	6
Understand the place value system	100	53	5 (major)
Use equivalent fractions as a strategy to add and subtract fractions	100	63	5 (major)
Use the four operations with whole numbers to solve problems	95	46	4
Something else	4	-	
None of these	-		
Not sure		1	

Grade 6 (n=110) *Level: major, supporting, additional*	**Q51** **Will Teach**	**Q52** **Most Time**	**Grade Level (type of cluster)**
Apply and extend previous understandings of multiplication and division to divide fractions by fractions	92	43	6 (major)
Apply and extend previous understandings of numbers to the system of rational numbers	89	36	6 (major)
Compute fluently with multi-digit numbers and find common factors and multiples	89	27	6 (additional)
Convert like measurement units within a given measurement system	66	8	5
Develop understanding of statistical variability	71	4	6 (additional)
Investigate chance processes and develop, use, and evaluate probability models	36	1	7
Reason about and solve one-variable equations and inequalities	91	56	6 (major)
Represent and analyze quantitative relationships between dependent and independent variables	74	16	6 (major)
Solve real-world and mathematical problems involving area, surface area, and volume	85	30	6 (supporting)
Understand and apply the Pythagorean Theorem	9	1	8
Understand ratio concepts and use ratio reasoning to solve problems	90	66	6 (major)
Understand the place value system	68	9	5
Use properties of operations to generate equivalent expressions	95	34	7
Use the four operations with whole numbers to solve problems	84	34	4
Something else	8	5	
None of these	1		
Not sure		8	

Grade 7 (n=120) *Level: major, supporting, additional*	**Q51** **Will Teach**	**Q52** **Most Time**	**Grade Level (type of cluster)**
Analyze proportional relationships and use them to solve real-world and mathematical problems	94	77	7 (major)
Apply and extend previous understandings of multiplication and division to divide fractions by fractions	89	30	6

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers	92	60	7 (major)
Define, evaluate, and compare functions	46	13	8
Draw, construct, and describe geometrical figures and describe the relationships between them	65	5	7 (additional)
Investigate chance processes and develop, use, and evaluate probability models	68	10	7 (supporting)
Perform arithmetic operations on polynomials	31	10	HS (algebra)
Solve real-life and mathematical problems involving angle measure, area, surface area, and volume	86	30	7 (additional)
Solve real-life and mathematical problems using numerical and algebraic expressions and equations	93	77	7 (major)
Summarize and describe distributions	53	7	6
Understand and apply the Pythagorean Theorem	24	4	8
Use equivalent fractions as a strategy to add and subtract fractions	83	16	5
Use properties of operations to generate equivalent expressions	90	38	7 (major)
Use random sampling to draw inferences about a population	68	1	7 (supporting)
Something else	11	5	
None of these	1		
Not sure		2	

Grade 8 (n=121) *Level: major, supporting, additional*	**Q51** **Will Teach**	**Q52** **Most Time**	**Grade Level (type of cluster)**
Analyze and solve linear equations and pairs of simultaneous linear equations	96	88	8 (major)
Analyze proportional relationships and use them to solve real-world and mathematical problems	84	30	7
Define, evaluate, and compare functions	90	55	8 (major)
Define trigonometric ratios and solve problems involving right triangles	18	1	HS (geometry)
Investigate patterns of association in bivariate data	71	11	8 (supporting)
Know that there are numbers that are not rational, and approximate them by rational numbers	89	13	8 (supporting)
Perform arithmetic operations on polynomials	59	18	HS (algebra)
Rewrite rational functions	44	4	HS (algebra)
Solve real-world and mathematical problems involving area, surface area, and volume	74	8	6
Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres	81	9	8 (additional)
Understand and apply the Pythagorean Theorem	94	27	8 (major)
Understand congruence and similarity using physical models, transparencies, or geometry software	72	17	8 (major)
Use properties of operations to generate equivalent expressions	87	39	7
Work with radicals and integer exponents	86	18	8 (major)
Something else	11	6	
None of these	-		
Not sure		4	

Q53. If you had to choose just one of these, do you think the CCSS-M cause greater changes in:

52 The way teachers teach
34 The content teachers cover
9 Something else (volunteered responses: both, the way students learn, the way students feel about math, extra work for teachers, reading and literacy in math, testing)
6 Not sure

Q54–62. How close does each of the following statements come to your view on the impact of the CCSS-M in the classroom?

% responding	NET Very/ Some-what	Very Close	Some-what Close	NET Not Too / Not at All	Not Too Close	Not Close at All	Not Sure
[Q54] Focusing classroom time and energy on fewer topics in more depth is leading to better practical understanding of math and its uses	**67**	32	35	**28**	15	13	5
[Q55] Games and exploration may seem to waste time, but they are effective for fostering deeper understanding of new math concepts	**67**	29	37	**29**	18	11	5
[Q56] Important math concepts are being missed because fewer topics are being covered	**28**	9	19	**68**	27	41	4
[Q57] The math materials available to me are not aligned with the standards	**42**	17	24	**55**	19	37	3
[Q58] Requiring written explanation for simple math problems is unnecessary and detracts from real learning	**30**	11	20	**66**	25	42	4
[Q59] The standards take away from the creativity and joy of teaching	**44**	21	23	**53**	23	30	3
[Q60] Students are developing a stronger capacity to persevere in math and come up with solutions on their own	**58**	22	35	**39**	23	16	4
[Q61] Students are developing a stronger number sense and more ability to apply math in real-world situations	**64**	28	36	**33**	20	13	3
[Q62] Students are frustrated because they're being asked to learn many different ways to solve the same problem	**53**	23	31	**44**	25	20	3

Q63–68. How close does each of the following statements come to your view on the broader impact of the CCSS-M beyond the classroom?

% responding	NET Very/ Some-what	Very Close	Some-what Close	NET Not Too / Not at All	Not Too Close	Not Close at All	Not Sure
[Q63] Expectations are unrealistic; there will be too many students unable to reach these standards	**53**	24	29	**43**	26	17	4
[Q64] For all the attention they are getting, these standards are very similar to the old ones	**62**	19	43	**34**	23	11	4
[Q65] Reinforcement of math learning at home is declining because parents don't understand the way math is being taught	**85**	53	31	**13**	9	4	3
[Q66] Students are getting better prepared for the advanced math needed to succeed in selective colleges or as STEM majors	**53**	20	33	**34**	23	11	14
[Q67] Teachers' math content knowledge will improve	**62**	29	33	**32**	19	13	6
[Q68] The standards will help ensure that America's young people have the math skills needed to compete in the global economy	**55**	22	33	**36**	22	14	9

Q69. For how many years have you been a full-time classroom teacher?

Mean 15.7 years

13 1–4
16 5–9
29 Less than 10
72 10 or more
42 10–20
29 21 or more

Q70–72.

% responding	Yes	No
[Q70] Do you have a major or minor in math or a related field, or not?	35	65
[Q71] Do you have any special certification or training in math, or not?	43	57
[Q72] Do you currently serve as chairperson of the math department or of a math committee, or not?	18	82

Q73. Approximately what percentage of students at your school qualify for free or reduced-price lunch?

18 0–24%
24 25–49%
26 50–74%
32 75–100%

Q74. Approximately what percentage of students at your school are African American and/or Hispanic?

46 0–24%
21 25–49%
17 50–74%
16 75–100%

Sex of survey respondents

13 Male
88 Female

Endnotes

1 T. Kane et al., "Teaching Higher: Educators' Perspectives on Common Core Implementation" (Cambridge, MA: Harvard University Center for Education Policy Research, February 2016), http://cepr.harvard.edu/files/cepr/files/teaching-higher-report.pdf?m=1456158749.

2 V. Darleen Opfer, et al., "Implementation of K–12 State Standards for Mathematics and English Language Arts and Literacy: Findings from the American Teacher Panel" (Santa Monica, CA: RAND Corporation, 2016), http://www.rand.org/pubs/research_reports/RR1529.html.

3 D. Rentner and N. Kober, "Common Core State Standards in 2014: Curriculum and Professional Development at the District Level" (Washington, D.C.: Center on Education Policy, October 29, 2014), http://cep-dc.org/displayDocument.cfm?DocumentID=441.

4 A sampling of titles from Dr. Bay-William's publications speaks to the practicality of her work: *Order of operations: The myth and the math; Three steps to mastering multiplication facts; Assessing basic facts fluency; Supporting Math Vocabulary; What does algebraic thinking look like?;* and *What parents want to know about standards-based mathematics curricula.* The book series is called "Elementary and Middle School Mathematics: Teaching Developmentally and the Teaching Student-Centered Mathematics."

5 http://www.rand.org/pubs/research_reports/RR1529.html

6 The non-adopting states, which were excluded from the survey, either failed to adopt or later rejected the CCSS-M. Still, it is difficult to gauge fidelity to the CCSS-M even in the "Common Core states." For example, South Carolina revised the language of the standards, but not the sequence of topics. (See South Carolina Department of Education, "South Carolina College- and Career-Ready Standards for Mathematics," 2015, http://ed.sc.gov/instruction/standards-learning/mathematics/standards/scccr-standards-for-mathematics-final-print-on-one-side/.) Similarly, states such as Tennessee and Florida have "revisited" the CCSS-M, but it's not clear if they have made any substantive changes.

7 The CCSS-M were released in June 2010.

8 Of course, choice of curricula may not be entirely up to the teacher. Survey results also show that 51 percent of teachers who have a primary textbook and 49 percent of those who are provided instructional materials say that they are "required" to use them (see Table 9). Teachers in the lower grades are far more likely to be subject to these mandates than those in middle school.

9 The Standards for Mathematical Practice are part of the CCSS-M. According to the standards website, they "describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important 'processes and proficiencies' with longstanding importance in mathematics education." (See http://www.corestandards.org/Math/Practice/.)

10 See "Common Core State Standards for Mathematics (CCSS-M)," http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf and "Key Instructional Shifts of the Common Core State Standards for Mathematics," http://achievethecore.org/content/upload/Focus%20in%20Math_091013_FINAL.pdf.

11 CCSS-M, http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf, 3.

12 At the high school level, they are grouped into content categories such as algebra and geometry.

13 S. Dingman et al., "Common Mathematics Standards in the United States: A Comparison of K–8 State and Common Core Standards," *Elementary School Journal* 113, no. 4 (June 2013), http://www.jstor.org/stable/10.1086/669939?seq=1#page_scan_tab_contents, 541–564.

14 L. Cogan et al., "Implementing the Common Core State Standards for Mathematics: What We Know about Teachers of Mathematics in 41 States," Working Paper 33 (East Lansing, MI: Michigan State University, January 2013), *http://files.eric.ed.gov/fulltext/ED558137.pdf*.

15 J. Davis et al., "Common Core State Standards for Mathematics: Middle School Mathematics Teachers' Perceptions" (Rochester, NY: University of Rochester, Warner School for Education, 2013), https://www.warner.rochester.edu/files/warnercenter/docs/commoncoremathreport.pdf.

16 T. Kane et al., "Teaching Higher: Educators' Perspectives on Common Core Implementation" (Cambridge, MA: Harvard University Center for Education Policy Research, February 2016), http://cepr.harvard.edu/files/cepr/files/teaching-higher-report.pdf?m=1456158749.

17 V. Darleen Opfer, et al., "Implementation of K–12 State Standards for Mathematics and English Language Arts and Literacy: Findings from the American Teacher Panel" (Santa Monica, CA: RAND Corporation, 2016), http://www.rand.org/pubs/research_reports/RR1529.html.

18 Editorial Projects in Education Research Center (EPERC), "Findings from a National Survey of Teacher Perspectives on the Common Core" (Bethesda, MD: EPERC, 2013), http://www.edweek.org/media/epe_survey_teacher_perspctives_common_core_2013.pdf.

19 Teachers were asked if they taught math—not if they had a math certification or degree. Also, elementary teachers tend to teach multiple subjects, including (but not limited to) math.

20 The non-adopting states, which were excluded from the survey, either failed to adopt or later rejected the CCSS-M. Still, it is difficult to gauge fidelity to the CCSS-M even in the "Common Core states." For example, South Carolina revised the language of the standards, but not the sequence of topics. (See South Carolina Department of Education, "South Carolina College- and Career-Ready Standards for Mathematics," 2015, http://ed.sc.gov/instruction/standards-learning/mathematics/standards/scccr-standards-for-mathematics-final-print-on-one-side/.) Similarly, states such as Tennessee and Florida have "revisited" the CCSS-M, but it's not clear if they have made any substantive changes.

21 J. Zimba, "The Development and Design of the Common Core State Standards for Mathematics," *New England Journal of Public Policy* 26, no. 1 (2014), http://scholarworks.umb.edu/nejpp/vol26/iss1/.

22 Achieve the Core, "K–8 Publishers' Criteria for the Common Core State Standards for Mathematics," http://achievethecore.org/page/266/k-8-publishers-criteria-for-the-common-core-state-standards-for-mathematics.

23 Ibid.

24 Achieve the Core, “Mathematics: Focus by Grade Level,” http://achievethecore.org/category/774/mathematics-focus-by-grade-level.

25 Grade 2 teachers were shown a list of thirteen topics.

26 Most teachers were shown five major topics. However, grade 2 and grade 7 teachers were shown four, and grade 3 teachers were shown six.

27 One exception: Grade 1 included seven (not six) off-grade-level topics.

28 See Appendix B, Question 51 for complete findings.

29 Although some grades have as many as eight major topics, the list of topics teachers could choose from included at most six major topics from their grade level.

30 See Appendix B, Question 52 for complete findings.

31 Technically, 89 percent of teachers reported teaching this cluster, not 90 percent.

32 See, for instance, CCSS-M, “Grade 3 >> Operations and Algebraic Thinking >> Multiply and Divide within 100 >>7: Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers,” http://www.corestandards.org/Math/Content/3/OA/C/7/.

33 University of Arizona Institute for Mathematics and Education, “Progression Documents for the Common Core Math Standards,” http://math.arizona.edu/˜ime/progressions/.

34 The numbers of teachers reporting that they taught above- or below-grade-level students is too small to detect a statistical relationship between it and teaching above- and below-grade-level clusters.

35 In two instances, the survey data show at least two grade levels above content taught by at least half of teachers. Specifically, 50 percent of grade 1 teachers report teaching a grade 3 cluster (“Develop understanding of fractions as numbers”). Sixty-seven percent of kindergarten teachers also report teaching a grade 2 cluster (“Measure and estimate lengths in standard units”).

36 The data include kindergarten teachers, although in fact kindergarten teachers are not expected to teach multi-step word problems.

37 Kindergarten and grade 8 teachers are not represented in the table because there is not a cluster focused on procedures at either grade. In grade 8, students are heavily engaged in applications; however, the focus is not on procedures but on algebra and geometry.

38 To the extent that CCSS-M have introduced more challenging math problems, these data may be somewhat confounded by problem difficulty. If problems are indeed more challenging, that would tend to depress the “more” percentage relative to “fewer” and “about the same.”

39 D. Rentner and N. Kober, “Common Core State Standards in 2014: Curriculum and Professional Development at the District Level” (Washington, D.C.: Center on Education Policy, October 29, 2014), http://cep-dc.org/displayDocument.cfm?DocumentID=441.

40 M. Polikoff, "How Well Aligned Are Textbooks to the Common Core State Standards in Mathematics?" *Education Policy Research Journal* (May 2015), http://aer.sagepub.com/content/early/2015/05/05/0002831215584435.abstract; B. Herold and M. Molnar, "Research Questions Common-Core Claims by Publishers," *Education Week*, March 3, 2014, http://www.edweek.org/ew/articles/2014/03/05/23textbooks_ep.h33.html?tkn=YOUFERL5a75ip%2Bu4Ekae832M1roz6xfajtmp&cmp=ENL-II-NEWS2.

41 Related, a recent RAND study (2016) found that 62 percent of elementary math and 41 percent of secondary math teachers reported using materials that their districts required.

42 Consistency in using instructional materials can be viewed positively or negatively, depending in part on if those materials are aligned to the standards. Further, several have voiced concerns that a "common" curriculum may be harmful to advanced learners if treated as an instructional ceiling. See J. Plucker, "Common Core and America's High-Achieving Students" (Washington, D.C.: Thomas B. Fordham Institute, February 22, 2015), http://edexcellence.net/publications/common-core-and-americas-high-achieving-students.

43 See, for example: CCSS.MATH.CONTENT.2.MD.B.6, CCSS.MATH.CONTENT.3.MD.A.1, CCSS.MATH.CONTENT.4.MD.A.2, CCSS.MATH.CONTENT.4.NF.C.6, CCSS.MATH.CONTENT.5.MD.B.2, CCSS.MATH.CONTENT.6.NS.C.6, CCSS.MATH.CONTENT.6.NS.C.7, CCSS.MATH.CONTENT.6.SP.B.4, CCSS.MATH.CONTENT.6.EE.B.8, CCSS.MATH.CONTENT.7.NS.A.1, CCSS.MATH.CONTENT.8.NS.A.2.

44 See 4.NBT.B.4, 5.NBT.B.5, 6.NS.B.2, 6.NS.B.3.

45 In grade 8, three of the top five were major grade-level topics.

46 Common Core State Standards Initiative, "Mathematics Standards," http://www.corestandards.org/Math/.

47 J. Zimba, "Can Parents Help with Homework? Yes." (Washington, D.C.: Thomas B. Fordham Institute, January 15, 2016), http://edexcellence.net/articles/can-parents-help-with-math-homework-yes.

48 Achieve the Core, "Publishers' Criteria." As described in the document: "This document, developed by the CCSS-M writing team with review and collaboration from partner organizations, individual experts, and districts using the criteria, aims to support faithful CCSS-M implementation by providing criteria for materials aligned to the Common Core State Standards for Mathematics."

49 Some authors, such as JUMP math's founder John Mighton, explicitly design their curricula to bring below-grade-level students into grade-level material. See http://www.jumpmath.org/jump/en/john_mighton.

50 For starters, teachers need help making decisions about which teaching strategies to use and in what order, as well as better understanding the CCSS-M content that comes before and after their grade/course.

51 Common Core State Standards Initiative, "Standards in Your State," http://www.corestandards.org/standards-in-your-state/.

52 Challenges exist relative to administering online surveys of teachers, particularly the difficulty of reaching teachers' email in-boxes, given the strong firewalls and SPAM filters that many school districts have in place.