State Mathematics Standards

An Appraisal of Math Standards in 46 States, the District of Columbia, and Japan

March 1998

By Ralph A. Raimi and Lawrence S. Braden

THOMAS B. FORDHAM FOUNDATION
OUTSIDE THE BOX

An Entry from the Diaries of John Adams (1735-1826)

June 1, 1756.

Drank Tea at the Majors. The Reasoning of Mathematics is founded on certain and infallible Principles. Every Word they Use, conveys a determinate Idea, and by accurate Definitions they excite the same Ideas in the mind of the Reader that were in the mind of the Writer. When they have defined the Terms they intend to make use of, they premise a few Axioms, or Self evident Principles, that every man must assent to as soon as proposed. They then take for granted certain Postulates, that no one can deny them, such as, that a right Line may be drawn from one given Point to another, and from these plain simple Principles, they have raised most astonishing Speculations, and proved the Extent of the human mind to be more spacious and capable than any other Science.

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EXPERT ADVISORY COMMITTEE

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FOREWORD

The Thomas B. Fordham Foundation is pleased to present this appraisal of state mathematics standards by Ralph A. Raimi of the University of Rochester and Lawrence S. Braden of St. Paul's School.

This is the fourth such publication by the Foundation. In July 1997, we issued Sandra Stotsky's evaluation of state English standards. In February 1998, we published examinations of state standards in history and geography. Science follows.

Thus, we will have gauged the states' success in setting standards for the five core subjects designated by the governors and President Bush at their 1989 education "summit" in Charlottesville. The national education goals adopted there included the statement that, "By the year 2000, American students will leave grades four, eight, and twelve having demonstrated competency in challenging subject matter including English, mathematics, science, history and geography." Although other subjects have value, too, these five remain at the heart of the academic curriculum of U.S. schools.

All are critically important, to be sure, but mathematics has special significance in today's debates about boosting the performance of U.S. students by setting ambitious standards for their academic achievement.

Mathematics is, of course, the third of the "three R's." Practically nobody doubts its central place in any serious education, its intellectual significance, or its practical value. Math is ordinarily the second subject (after reading) that young children encounter in primary school. "Math aptitude" constitutes half of one's S.A.T. score. And it was in no small part the weak math performance of American youngsters on domestic and international assessments that led us to understand that the nation was at risk. (Because it is universal, because it is sequential and cumulative, and because its test questions are easy to translate, mathematics has long been the subject most amenable to illuminating cross-national comparisons of student performance.)

Math also blazed a trail into the maze of national standards. Even as the Charlottesville summit was convening, the National Council of Teachers of Mathematics (NCTM) was putting the finishing touches on its report entitled *Curriculum and Evaluation Standards for School Mathematics.* In the ensuing decade, that publication and its progeny have had considerable impact on U.S. education, not least on the state math standards reviewed in the following pages. I have no doubt that, of all the "national standards" set in the various academic subjects, these have been the most influential. Indeed, I have heard policy makers declare that what America needs in other academic subjects are counterparts to the "NCTM math standards."

It is vital to understand, however, that the NCTM's mission was not—and today is not—the codification of traditional school mathematics into clear content and performance standards. Rather, NCTM's main project was to transform the teaching and learning of mathematics in U.S. schools.

The effects of that hoped-for transformation on state math standards are abundantly clear in this appraisal. Some readers may judge that the states should go further still to transform their expectations for students and teachers in the direction set forth by NCTM. Others will judge that they have gone much too far already. In any case, it's noteworthy that today, nine years after it was unveiled, "NCTM math" no longer commands the public consensus that it once appeared to have. California, for example, recently adopted new statewide standards that could fairly be termed a repudiation of the NCTM approach.

The important thing to know about the present document is that we did not ask its authors—a distinguished university mathematician and a deeply experienced school math teacher—to grade the states on how faithfully their standards incorporate the NCTM's model for math education. Rather, we asked them to appraise state standards in terms of their own criteria for what excellent math standards should contain.

Advised by two other nationally respected scholars, the authors did precisely that. They developed nine criteria (under four headings) and then applied them with great care to the math standards of 46 states and the District of Columbia. (The remaining four states either do not have published standards or would not make their current drafts available for review.) For comparison purposes, the authors also describe Japan's math standards and apply their criteria to these.

The results are sobering. Only three states (California, North Carolina, and Ohio) earn "A" grades, and just nine get "B's." Those 12 "honor" grades must be set alongside 16 failing marks (and seven "C's" and 12 "D's").

The results differ markedly from those of the recent Council for Basic Education (CBE) appraisal of the "rigor" of state math standards at grades 8 and 12. The CBE study begins with a list of performance standards expressed in 51 clauses (or "benchmarks") for the 8th grade and 30 for the 12th. These clauses are largely drawn from the NCTM standards of 1989. The state documents under study were then scanned for those 81 demands, which, when present (and weighted by their closeness to the template clauses), were counted up for a total score.

The present document does not begin with a list of this kind, and similarity to the NCTM standards was not a

desideratum. The criteria used by Braden and Raimi are well described within the report itself, and include not only analyses of the "academic content" expressed or implied, but also qualities of exposition and taste affecting the standards' usefulness.

In view of the ferment in American math education and the continuing lackluster performance of U.S. youngsters in this key discipline, we must take notice of the findings reported herein. While state math standards are in many cases too new for them fairly to be held responsible for pupil attainment in this discipline, it appears that these documents, which were supposed to improve the situation, in most cases will not help and in many instances appear to be symptoms of the very failure they were intended to rectify.

To be sure, excellent math education continues in some classrooms and schools. State standards are not supposed to place a ceiling on how much is taught and learned. But they are meant to serve as a floor below which schools and teachers and children may not sink. As we learn from Messrs. Raimi and Braden, in many states today that floor seems to have been confused with the muddy excavation that ordinarily precedes construction.

We are grateful indeed to both authors for the rare energy, thoroughness, and mathematical insight that they brought to this arduous project. Raimi is professor emeritus of mathematics at the University of Rochester and former chairman of the math department (and graduate dean) at that institution. His scholarly specialty is functional analysis, and he has had a lifelong interest in effective mathematics teaching. Braden has taught mathematics and science in elementary, middle, and high schools for many years in Hawaii, in Russia, and now in New Hampshire. He is a recipient of the Presidential Award for Excellence in Science and Mathematics Teaching. He holds a bachelor's degree in mathematics from the University of California and an M.A.T. in mathematics from Harvard.

We also thank the two distinguished scholars who advised the authors throughout. Henry Alder is professor emeritus of mathematics at the University of California and a former president of the Mathematical Association of America. He has been a member of the California State Board of Education and recently served on the committee to rewrite that state's mathematics framework. Harold Stevenson is professor of psychology at the University of Michigan, a 1997 recipient of the American Psychological Association's Distinguished Scientific Award, and can fairly be termed America's foremost authority on Asian primary/secondary education and its comparison with U.S. schools and students. Among many publications, he co-authored *The Learning Gap,* a pathbreaking analysis of elementary education in Asia and the United States. He has a particular interest in the standards, curricula, and pedagogy of mathematics, which discipline has been the focus of many of his comparative studies, and has been deeply involved with the Third International Mathematics and Science Study (TIMSS).

In addition to published copies, this report (and its companion appraisals of state standards in other subjects) is available in full on the Foundation's web site: http://www.edexcellence.net. Hard copies can be obtained by calling 1-888-TBF-7474 (single copies are free). The report is not copyrighted and readers are welcome to reproduce it, provided they acknowledge its provenance and do not distort its meaning by selective quotation.

For further information from the authors, readers can contact Ralph Raimi by writing him at the Department of Mathematics, University of Rochester, Rochester, N.Y. 14627, or e-mailing rarm@db2.cc.rochester.edu. Lawrence Braden can be e-mailed at lbraden@sps.edu. The Thomas B. Fordham Foundation is a private foundation that supports research, publications, and action projects in elementary/secondary education reform at the national level and in the vicinity of Dayton, Ohio. Further information can be obtained from our web site or by writing us at 1015 18th Street N.W., Suite 300, Washington, D.C. 20036. (We can also be e-mailed through our web site.) In addition to Messrs. Raimi and Braden and their advisors, I would like to take this opportunity to thank the Foundation's program manager, Gregg Vanourek, as well as staff members Irmela Vontillius and Michael Petrilli, for their many services in the course of this project, and Robert Champ for his editorial assistance.

Chester E. Finn, Jr. President
Thomas B. Fordham Foundation
Washington, D.C.
March 1998

EXECUTIVE SUMMARY

Almost every one of the 50 States and the District of Columbia have by now published standards for school mathematics, designed to tell educators and the public officials who direct their work what ought to be the goals of mathematics education from kindergarten through high school. They are generally given as "benchmarks" of desired achievement as students progress through the grade levels to graduation, though sometimes they include guides to pedagogy as well. The present report represents a detailed analysis of all such documents as were available, 47 in all, though it has only space to offer rather abbreviated judgment of their value and a rating of their comparative worth. Grades of A, B, C, D, or F were given to each state, based on an analysis of the contents according to criteria and grade levels as described early in the report. Some comments on each State conclude the report.

On the whole, the nation flunks. Only three states received a grade of A, and just nine others a grade of B.

On the whole, the nation flunks. Only three states received a grade of A, and just nine others a grade of B. More than half receive grades of D or F, and must be counted as having failed to accomplish their task. The grading is described below, but it should be understood that anything less than an A should be unacceptable.

A state, after all, is not a child to be graded for promise or for effort; the failure of a state to measure up to the best cannot be excused for lack of sleep the night before the exam. The failure of almost every State to delineate even that which is to be *desired* in the way of mathematics education constitutes a national disaster.

The failure of almost every State to delineate even that which is to be *desired* in the way of mathematics education constitutes a national disaster.

Even if the states' standards documents were exemplary, there would remain a problem of implementation. The public usually hears of the problems of schools as questions of funding, of discipline, and even sometimes of teacher preparation or recruitment, but it generally imagines that their intellectual goals are clear. For basketball players and musicians the goals are indeed known. But for elementary and secondary education in the United States today, there are no such agreements in place regarding its essential core: its academic program. This is especially so in mathematics, as the standards under review here illustrate.

The authors of this report believe it unconscionable that, in writing these standards—these documents of pure *intent*, whose success depends only on the efforts of experts already in place—so many states are so remiss in their duty.

As we have seen it, the principal failures stem from the mathematical ignorance of the writers of these standards, sometimes compounded by carelessness and sometimes by a faulty educational ideology. We are convinced that the average math teacher can be led to a better grasp of both the material that should be taught at various grade levels and the manner in which it should be presented, than the writers and editors of these documents imagine.

Our criteria for judgment were four: Clarity of the document's statements, and sufficient Content in the curriculum described or outlined in the text, were our first two demands. Third, since deductive reasoning is the backbone of mathematics, we looked to see how insistently that quality (denominated Reason) was to be found threaded through all parts of the curriculum. Finally, we assessed whether the document avoided the negative qualities that we called False Doctrine and Inflation. These four major criteria, some of them broken down into sub-criteria, were individually graded and the scores combined for a single total.

The most serious failure was found in the domain of Reason. There is visible in these documents a currently fashionable ideology concerning the nature of mathematics that is destructive of its proper teaching. That is, mathematics is today widely regarded (in the schools) as something that must be presented as usable, "practical," and applicable to "real-world" problems at every stage of schooling, rather than as an intellectual adventure.

Mathematics does indeed model reality, and is miraculously successful in so doing, but this success has been accomplished by the development of mathematics itself into a structure that goes far beyond obvious daily application. Mathematics is a deductive system, or a number of such systems related to one another and to the world, as geometry and algebra are related to each other as well as to statistics and physics; to neglect the systematic features of mathematics is to condemn the student to a futile exercise in unrelated rule memorization. Most of the standards

documents we have read, for all that they claim to foster "understanding" above rote learning, lack the qualities that would lead their readers, America's teachers, in the desired direction.

This lack of logical progression, seen especially in what passes for geometry and algebra in the grades from middle school upward, is visible in the lack of clarity of the documents. It is also visible in their advocacy of the use of calculators and computers in the early grades, where arithmetic and measurement as ideas should rather be made part of the student's outlook, by his learning through much experience and practice the nature of the number system. Learning to calculate, especially with fractions and decimals, is more than "getting the answer"; it is an exercise in reason and in the nature of our number system, and it underlies much that follows later in life. Only a person ignorant of all but the most trivial uses of calculation will believe that a calculator replaces—during the years of education—mental and verbal and written calculation. Adults have need of calculators, and indeed computer programs, for computing their income taxes and doing their jobs. But the educational needs of children are quite different.

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Content was the most successful part of these documents. This country has a traditional curriculum from the point of view of content, and many states at least mention most of it, including such recent additions as statistics and probability. However, much has been lost, especially from the Euclidean geometry that was so large a part of a high school program 50 years ago; and the fragmentation of the curriculum into too many different "threads" has also diluted the traditional curriculum.

The enterprise of writing standards goes hand-in-hand with the improvement of classroom practice, and there is no doubt that teachers of the next few years, seeing the inadequacy of most of what we have surveyed, will themselves offer suggestions for improvement. Members of the public, too, are often dissatisfied with vague education, led by vague standards, and they, too, will be heard. We believe the exercise of writing these documents is worthwhile, and we wish more states took it seriously enough to put their best talent to work on them.

In particular, the "best talent" must include not only members of the school establishment and state departments of education, but also persons knowledgeable in the uses of mathematics and the creation of new mathematics. That is to say, scientists (including statisticians, engineers, and applied mathematicians) and research mathematicians from the mathematics departments of the universities. These two communities have been most noticeably absent from the first rounds of standards construction, and future improvement is not possible without them.

NATIONAL REPORT CARD
State Math Standards

State (in alphabetical order)	Score	Grade
Alabama	11.5	B
Alaska	7.5	C
Arizona	12.2	B
Arkansas	2.5	F
California	16.0	A
Colorado	5.4	D
Connecticut	4.7	D
Delaware	7.8	C
District of Columbia	4.7	D
Florida	4.8	D
Georgia	11.5	B
Hawaii	1.3	F
Idaho	2.2	F
Illinois	6.8	D
Indiana	8.3	C
Iowa	—	N
Kansas	4.5	D
Kentucky	5.2	D
Louisiana	1.3	F
Maine	3.5	F
Maryland	3.0	F
Massachusetts	1.3	F
Michigan	1.3	F
Minnesota	—	N
Mississippi	11.9	B
Missouri	2.7	F
Montana	1.5	F
Nebraska	3.0	F
Nevada	—	N
New Hampshire	6.2	D
New Jersey	9.2	C
New Mexico	2.5	F
New York	11.3	B
North Carolina	14.2	A
North Dakota	5.6	D
Ohio	13.5	A
Oklahoma	3.9	F
Oregon	5.3	D
Pennsylvania	5.5	D
Rhode Island	1.0	F
South Carolina	4.5	D
South Dakota	1.5	F
Tennessee K-8	2.5	F*
Tennessee 9-12	14.0	A*
Tennessee average	8.25	C
Texas	10.6	B
Utah	11.7	B
Vermont	8.3	C
Virginia	11.8	B
Washington	2.5	F
West Virginia	12.5	B
Wisconsin	7.4	C
Wyoming	—	N
Japan	15.0	A

State (by rank)	Score	Grade
California	16.0	A
Japan	15.0	A
North Carolina	14.2	A
Tennessee 9-12	14.0	A*
Ohio	13.5	A
West Virginia	12.5	B
Arizona	12.2	B
Mississippi	11.9	B
Virginia	11.8	B
Utah	11.7	B
Alabama	11.5	B
Georgia	11.5	B
New York	11.3	B
Texas	10.6	B
New Jersey	9.2	C
Indiana	8.3	C
Vermont	8.3	C
Tennessee average	8.25	C
Delaware	7.8	C
Alaska	7.5	C
Wisconsin	7.4	C
New Hampshire	7.2	C
Illinois	6.8	D
North Dakota	5.6	D
Pennsylvania	5.5	D
Colorado	5.4	D
Oregon	5.3	D
Kentucky	5.2	D
Florida	4.8	D
Connecticut	4.7	D
District of Columbia	4.7	D
Kansas	4.5	D
South Carolina	4.5	D
Oklahoma	3.9	F
Maine	3.5	F
Maryland	3.0	F
Nebraska	3.0	F
Missouri	2.7	F
Arkansas	2.5	F
New Mexico	2.5	F
Tennessee K-8	2.5	F*
Washington	2.5	F
Idaho	2.2	F
Montana	1.5	F
South Dakota	1.5	F
Hawaii	1.3	F
Louisiana	1.3	F
Massachusetts	1.3	F
Michigan	1.3	F
Rhode Island	1.0	F
Iowa	–	N
Minnesota	–	N
Nevada	–	N
Wyoming	–	N

Grading Scale: A= 13 – 16, B = 10 – 12.9, C = 7 –9.9, D = 4 – 6.9, F = 0 – 3.9, N = Not Evaluated

* Partial Grades

I. THE SCOPE OF THIS STUDY: THE DOCUMENTS THEMSELVES

The authors were commissioned by the Thomas B. Fordham Foundation to study and compare the "standards" or "frameworks" for school mathematics as published by the 50 States and the District of Columbia, and to grade them according to a set of uniform criteria representing the several purposes or qualities such documents should have.

An initial judgment was needed even to solicit the correct publication, and we began by sending 51 letters, identical except as to the addressee. The text of the letter defines what sort of state publication we intended to study. Here is one of them:

Department of Mathematics
UNIVERSITY OF ROCHESTER
Rochester, New York 14627
Tel (716) 275-4429 or 244-9368 • FAX (716) 244-6631 or 442-3339
Email: RARM@db2.cc.rochester.edu

Ralph A. Raimi
Professor Emeritus

13 July 1997

State of Arkansas
Department of Education
4 State Capitol Mall
Little Rock, Arkansas 72201-1071

Office of the Superintendent or Director:

I am conducting for the Fordham Foundation (1015 18th St. NW, Suite 300, Washington, D.C. 20036) a state-by-state analysis of the mathematics curriculum and assessment standards of those States that publish a formal, public Framework or Standards for mathematics at the levels K-12. I write this letter to ask you to send me what your state publishes of this nature.

Since the 50 States (and D.C.) have different ways of going about these matters it is hard for me to name the documents I want. I know that in California, for example, they have a "framework," while New York publishes separate pamphlets with titles such as "Three Year Sequence for High School Mathematics (I, II, and III)," i.e., a detailed curriculum guide as specified by the Regents. Other states might have a set of sample examinations by grade, which define the content by implication.

However you do it, I am interested in the guidelines by which local school districts in your state know what the state education department considers appropriate (or mandatory, if such is the case) in mathematics instruction and testing at all levels from kindergarten to high school graduation.

If there is a charge for these documents, please let me know and I will send in payment a check made out according to your instructions.

If there is some other office I should address this request to, please either forward my letter there or let me know the proper office to write to, or to phone or fax.

You can communicate with me by telephone, fax or email; all relevant addresses are found on the letterhead above.

Thank you for your attention.

Sincerely yours,
<signed>
Ralph A. Raimi

About 30 states and the District of Columbia responded by sending printed or photocopied documents. Three or four states invited us to download such a document from a Web page of the state's education department, and a few replied explaining that the standards (or frameworks) documents we wanted were in the process of revision and not available. (The District of Columbia will hereafter be included as a "state" when we speak of the states generically.) Alaska wrote saying it printed such a document but does not send it out of state, so we secured a copy from a friend who lives there. We later discovered that Iowa, which did not respond at all, does not publish anything answering our description. In most other cases of non-response we nonetheless located a Web page containing the document freely available to the public, and we obtained it. During the writing of this report, other states completed and sent us their latest drafts, or published them on their Web pages, but Minnesota, Nevada, and Wyoming do not have their current drafts in quotable form. Other states having their standards in draft form are included in this study, though we have made clear that they are drafts, not yet adopted. In all, we have ended with documents from 46 states and the District of Columbia, and for comparative purposes one similar document from Japan. All states are named in the Appendix to this report, where enough bibliographical information is given to make clear just what we used for our commentary and where it was obtained, or by what state office published, and when. We hope we have not included any document that has been superseded by the time of publication of this report. Of this, however, we cannot be certain.

It can be seen from the variety of titles in the Appendix that there is no uniform nomenclature, the words "Standards" and "Frameworks" often being used more or less interchangeably, along with "Curriculum Content Guide," etc. Certain States do define a difference. Alaska, California, Colorado, New Jersey, Pennsylvania and South Carolina are among those clear in this regard, publishing one document called *Standards* and another called *Framework*. In this usage, a "framework" will usually include the "Content Standards" or refer to it as a subsidiary document, but will also include guidelines for pedagogy, assessment, and other structural features of the mathematics program. In cases where a second document (usually a "framework") is in our possession but contributes nothing to what we are judging, we may omit its mention in the Appendix. Where the distinction or dual publication makes a difference in the interpretation of our results, the second document will be listed. Also, some of the standards we received are printed in company with standards for science and other subjects; we shall of course be reporting only on the mathematics sections, and such remarks introductory to the whole document as are relevant to mathematics.

Except where we are actually naming a document for reference we shall refer to all these publications generically as "standards." Whatever they are named, they are either the state's official answer to the description given in our letter of solicitation reprinted above, or what we construe as such an answer from the state's publications on its Internet web pages. As will appear in the description of the criteria we used to judge these documents, the presence of non-curricular information in documents labeled "framework" will make no difference in our ratings, since pedagogical and assessment advice and procedures are not part of what we consider in this study, except as their presence might cast additional light on the content (and sometimes "performance") standards that are the real subject of this report.

II. WHAT STANDARDS ARE

Standards are intended as a statement of what students should learn, or what they should have accomplished, at particular stages of their schooling. They are thus inferentially a guide to curricula, instruction and examinations, but are always something less than a curriculum outline by which detailed textbooks or examinations could be completely envisioned. They are also less than complete guides to instruction in another way, in that they do not intend, except incidentally, to prescribe or suggest actual pedagogical procedure. Detailed lesson guides, examinations, and prescriptions as to pedagogy are in most states left to local school districts, or published separately from the standards document.

A few states do give statewide tests at certain grade levels, and have designed their standards documents accordingly, and it appears that more states are planning to do this in the future; even so, a typical state standards document is intended to be compatible with any of a number of different textbook choices and methods of instruction. It is only intended to assure a uniformity of outcome, as might be measured by a statewide examination if there was one. Where such examinations are not mandated, a state's standards are still intended as serious advice to the local school districts.

While the standards documents under review usually omit prescriptions concerning pedagogy, but rather attempt to state simply what should be known (however arrived at) by students at each stage of their progress, there are documents that do importantly include pedagogical advice. In a few cases this advice is considered by us despite the above disclaimers, but only to the degree that this extra information casts essential light on the meaning of the content standards themselves.

For example, a state might prescribe the use of calculators in the teaching of "long division," and with such firmness that we are forced to conclude that the usual "long division" algorithm is not, or is hardly, to be taught at that point. In such a case, while the standards might also state that 6th grade students (say) should "divide and multiply numbers in decimal notation," we cannot credit the document with demanding that proficiency with the standard algorithm is intended unless it is so stated. Similarly with sample examination questions, classroom scenarios, etc., which are not part of our assessment except as they clarify the content demands. Our own interest is in the curriculum itself, or (more accurately) its intended result, as dictated or implied by the standards as published.

III. THE STANDARDS MOVEMENT IN RECENT YEARS

The best known document of the sort under review is not a publication of a state at all, nor of the U.S. government. It is the 1989 publication of the National Council of Teachers of Mathematics (NCTM) called *Curriculum and Evaluation Standards for School Mathematics*, and generally referred to as "The NCTM Standards." Since 1989, this document has been the single most influential guide to changes in the nation's K-12 mathematics teaching, and in the contents and attitudes of our best-selling textbooks.

The most obviously missing voice in this listing of those influential in school mathematics today is that of the mathematics profession itself.

The NCTM is a non-governmental professional association, founded in 1920, which has become one of the principal voices of its profession. Other voices are the two teachers' unions (NEA and AFT), the 51 state education departments themselves, all of which all have specialists in school mathematics, the U.S. Department of Education and, less officially, the faculties and deans of the major schools of education.

The NCTM publishes a monthly magazine, *The Mathematics Teacher,* and also other, more specialized journals, including a journal of mathematical education research. Its membership consists almost entirely of school teachers, professors in university schools of education, and education administrators and officials at all levels, and far exceeds the combined membership of the professional mathematicians' organizations. NCTM also publishes many other guides, yearbooks, and research reports, and it conducts national and regional meetings at which professional information is exchanged.

The most obviously missing voice in this listing of those influential in school mathematics today is that of the mathematics profession itself, as it might be represented by the three major professional organizations: The American Mathematical Society (AMS), the Mathematical Association of America (MAA), and the Society for Industrial and Applied Mathematics (SIAM). The MAA is devoted to the advancement of the *teaching* of mathematics at the college level, whereas the other two are principally interested in scientific research and publication. All three take an interest in the health of the profession itself; but none of them has traditionally considered school education at the K-12 level an interest of more than marginal priority. While a few members of NCTM are professors of mathematics in university mathematics departments, it is doubtful that one in a hundred of those persons who call themselves "mathematician" or belong to the American Mathematical Society, also belongs to NCTM or reads *The Mathematics Teacher.*

Yet a serious attempt to bridge the gulf of interests and influence between the world of K-12 mathematics education and the world of professional mathematics was made during the era of "The New Math," roughly the period 1955-1970, the effective bridge then having been the curriculum-writing projects and "teachers' institutes" financed (mainly) by the National Science Foundation at the time. This passed without lasting success, and it appears that, while a few mathematicians have always been present at councils concerned with school mathematics, or in grassroots projects of educational experimentation, or in textbook-writing teams, and while as professors they have been among the teachers of those who later become school teachers and professors of education, their influence on curricula and classroom practice has been negligible over most of the present century, including today. Professors of education, even mathematics education, are members of a "second culture" almost as distinct from the world of mathematics as C.P. Snow's literary culture was from that of his friends the physicists.

This separation is a cultural phenomenon more pronounced in the United States than in most European countries and Japan, and it is not really a necessary concomitant of the different professional responsibilities of the two groups. Indeed, another "bridging" effort on the part of the mathematical societies and NCTM is being made today, especially in connection with the movement towards standards for school mathematics, but it is only beginning. The results will be impossible to assess for some time to come.

The 1989 NCTM Standards, which was the work of NCTM and not the mathematics profession, is comparable to almost all of the state "standards" volumes under study in this report, and in many cases is their acknowledged ancestor; but it is not completely so. It is much longer than any of them (except for the New Jersey "framework"), and contains much advice on pedagogy, including "vignettes," i.e., imagined dialogues or classroom conversations, illustrating the way NCTM expects its recommendations might play out in practice. Its influence is manifest in most state standards, even where they confine themselves to curricular content. Its educational philosophy, usually briefly and inadequately referred to as "constructivism," and its categories of mathematics curriculum: e.g., "mathematics as

problem-solving," "mathematics as communication," "mathematics as reasoning," "measurement," "number sense," etc., are all echoed, often strongly and by explicit cross-reference, in the state documents.

The NCTM Standards offers 13 such categories (each called a "standard") for Grades K-8, and 14 for Grades 9-12. Mostly the list is the same for the two levels, but there is some variation: Some titles which are applicable to K-8 (e.g., "whole number computation") may become obsolete at the 9-12 level, while a few appear at 9-12 ("trigonometry") that could not reasonably have been part of the earlier work. The educational philosophy of NCTM follows the maxim of Jerome Bruner, an influential psychologist of education, who said that there is no subject that cannot be presented to a child of any age whatsoever in some intellectually respectable form. From Bruner's psychological theories arises the doctrine of the "spiral curriculum," according to which learning is best constructed by the individual in stages, each subject returned to again and again, but each time at a higher "cognitive level."

Thus, "Problem-solving," "Geometry," and "Reasoning" are titles applicable to lessons at all levels. "Statistics," for another example, is now a staple of kindergarten mathematics, though exemplified mainly by exercises of a data-gathering sort. ("Find out the favorite ice-cream flavors of your classmates.") And the rubric "Algebra" can also find its way into the earliest grades under the alternate title of "patterns" (often construed quite literally).

A second list of rubrics for mathematical accomplishment in the schools is provided by the periodic National Assessment of Educational Progress (NAEP), in which K-12 mathematical progress is measured by a Federal agency under the headings: 1. Number Sense, Properties and Operations; 2. Measurement; 3. Geometry; 4. Data Analysis, Statistics and Probability; and 5. Algebra and Functions. Some states have organized their standards with attention to these categories.

IV. JUDGMENT, CRITERIA, AND THE RATINGS: AN OVERVIEW

Whether this or that *rubric* is represented at each grade level will be of little interest to the present report, in which we shall rather pay our attention to the *topics* that actually appear, and to what depth of knowledge or understanding is demanded in each appearance. Whether those topics we count appropriate appear under one or another psychological (or philosophical) rubric will not matter. This is fortunate, as the rubrics are not always strictly comparable among states. The age of the child is.

Teachers and school districts looking for practical guidance tend to ask what to teach in the 7th grade, for example, and what book to choose for that grade level, rather than to ask what sequence of topics in probability, say, should obtain in the K-12 progression as a whole, or whether their state succeeds in classifying what is accomplished in each grade into a list of independent rubrics covering mathematics comprehensively. It is, in fact, curious that the NCTM, and hence the states, have spent so much effort in this direction, whose principal consequence appears to have been the fragmentation of the mathematics curriculum during each year of schooling into a multitude of themes, or "threads," with consequent loss of depth in each, not many of Bruner's "higher cognitive levels" having been in fact reached by the spiraling. This phenomenon has been characterized by one knowledgeable commentator, concerning the United States' poor performance on the 1996 *Third International Mathematics and Science Survey* (TIMSS), who remarked that the typical American curriculum is "a mile wide and an inch deep." The same can be observed from our children's performance on the NAEP mathematics tests.

In contemplating each state's standards, we shall ignore most theoretical debates and simply ask the text to have this function: We imagine a newly arrived teacher in a remote district, of average knowledge and experience in teaching the mathematics traditionally taught at the grade level he works in. He has access to textbooks and other materials, but does not have previous experience in the schools of that particular state. He wishes to construct a syllabus and choose appropriate books (and perhaps technical tools, such as graph paper, compasses, computers, and calculators) in order to set up lesson plans and examinations.

This (imagined) teacher is not for this purpose asking the state for instruction in pedagogy, or even in mathematics; he only wants to know what his state *demands* the student should know of mathematics by grades 4, 8, and 12; or perhaps by end of kindergarten, grade 3, grade 6, grade 9 and grade 12; or even grade-by-grade.

The standards published by the state should answer this question.

Choosing textbooks and deciding the mixture of lecture, homework, group work, and classroom discussion are not usually, under the American tradition of local control, the decision of the state at all; but it has increasingly become the custom that the framework for results and even a list of the desired results themselves should be of statewide uniformity, at least approximately.

Our judgments of how the state has complied with this request, our ratings of the documents we call "standards," will be based on four major investigations:

1. Our estimate of the success the printed document will have in plainly telling this imagined teacher what he will need to know to satisfy the requirements of the state's citizens as to content;

2. Our estimate of how well the state's criteria, as we have been able to understand them, match our own standards of what schoolchildren should know, and our own experience of what they can be expected to know—and demonstrate knowledge of—by the grade levels named;

3. Our estimate of how well the document as a whole demands that the students' command of the "topics" taught and learned include the understanding of the *mathematical reasoning* and *unifying structures* that distinguish such mastery from the simple gathering of information; and

4. Our estimate of whether the document, whatever success it may otherwise enjoy in clarity and sufficiency of content, at the same time *injures its own purpose* by asking things it should not, or giving advice it should not, or by the exhibition of ignorance, carelessness, or pretension.

These four descriptions are summaries of our categories of judgment, but still need detailing: Just what properties should worthwhile standards display? We have isolated nine criteria for judgment, but, following the four points enumerated in the preceding section, we group them under the following four short titles:

(I) Clarity
(II) Content
(III) Reason
(IV) Negative Qualities

We shall explain these criteria in two stages: first, by giving them definitions (Section V); and secondly, by illustrative examples (Section VI). Following this exposition we shall present the ratings of the states in the form of a matrix of numerical grades (Section VII).

The general explanation of why a state receives a grade it does, under any of the criteria, will be implicit in our descriptions of the criteria. Section VII, the numerical scores, will nonetheless be introduced by a further explanation of our ratings. The table of the grades themselves (i.e., the Ratings), brief as it is, represents the goal, and is the condensation of the judgment of the authors after study of the documents in light of the criteria.

V. DEFINITIONS OF THE CRITERIA

I. Clarity refers to the success the document has in achieving its own purpose, i.e., making clear to the (imagined) provincial teacher what the state desires. Clarity refers to more than the prose. (Furthermore, some of the other evils of bad prose will have a negative category of their own in the fourth group, Negative Qualities.)

Is the State asking K-12 instruction in mathematics to contain the right things, and in the right amount and pacing?

(A) Firstly, of course, the words and sentences themselves must be understandable, syntactically unambiguous, and without needless jargon.

(B) Secondly, what the language says should be mathematically and pedagogically definite, leaving no doubt of what the inner and outer boundaries are, of what is being asked of the student or teacher.

(C) And thirdly, the statement or demand, even if understandable and completely defined, might yet ask for results impossible to test in the school environment, whether by a teacher's personal assessment or a statewide formal paper-and-pencil examination. We assign a positive value to testability.

Thus, the first group, Clarity, gives rise to three criteria:

I-A Clarity of the language
I-B Definiteness of the prescriptions given
I-C Testability of the lessons as described

II. Content, the second group, is plain enough in intent. Mainly, it is a matter of what might be called "coverage," i.e., whether the topics offered and the performance demanded at each level are sufficient and suitable. To the degree we can determine it from the standards documents, we ask, is the State asking K-12 instruction in mathematics to contain the right things, and in the right amount and pacing?

Here we shall separate the curriculum into three parts (albeit with fuzzy edges): Primary, Middle, and Secondary. It is common for states to offer more than one 9-12 curriculum, but also to print standards describing only the "common" curriculum, or one intended for a "school-leaving" examination in grade 11 or so. In such cases we shall not fault the document for failing to describe what it has no intention of describing, though we may enter a note mentioning the omission. Other anomalies will give rise to notes in Section VIII, which follows the actual ratings.

We cannot judge the division of content with year-by-year precision because very few states do so, and we wish our scores to be comparable across states. As for the fuzziness of the edges of the three divisions we do use, not even all those states with "elementary," "intermediate," and "high school" categories divide them by grades in the same way. One popular scheme is K-6, 7-9, and 10-12 for "elementary," "intermediate" and "high" schools, while another divides it K-5, 6-8, and 9-12. In cases where states divide their standards into sufficiently many levels (sometimes year-by-year), we shall use the first of these schemes. In other cases we will merely accept the state's divisions and grade accordingly. Therefore, Primary, Middle, and Secondary will not necessarily mean the same thing from one state to another. There is really no need for such precision in our grading, though of course in any given curriculum it does make a difference where topics are placed.

Thus, the second group, Content, gives rise to three criteria:

II-A Adequacy of Primary school content (K-6, approximately)
II-B Adequacy of Middle school content (or 7-9, approximately)
II-C Adequacy of Secondary school content (or 10-12, approximately)

In many states, mathematics is mandatory through the 10th grade, while others might vary this by a year or so. Our judgment of the published standards will not take account of what is or is not mandatory in each state; thus, a rating will be given for II-C whether or not all students in fact are exposed to part or all of it. (Some standards documents, as in the case of the District of Columbia, only describe the curriculum through grade 11, and we adjust our expectations of content accordingly.)

The difficult question here is to define "adequacy" of content. This can only be done by a listing of some sort. The authors of this report have in their minds a standard of what is possible and desirable to be taught at each level of school mathematics, and it is this standard of ours against which the adequacy of the states' standards is measured. To set this "standard" forth in abstract terms, by definition rather than enumeration, is, we believe, impossible. We

shall therefore attempt to do it by two rather indirect means, as will be explained in Section VI, Examples for the Criteria.

III. Reason, the third "group," will have only the one entry: Reason.

Civilized people have always recognized mathematics as an integral part of their cultural heritage. Mathematics is the oldest and most universal part of our culture, in fact, for we share it with all the world, and it has its roots in the most ancient of times and the most distant of lands.

The beauty and efficacy of mathematics both derive from a common factor that distinguishes mathematics from the mere accretion of information, or application of practical skills and feats of memory. This distinguishing feature of mathematics might be called *mathematical reasoning*, reasoning that makes use of the structural organization by which the parts of mathematics are connected to *each other*, and not just to the real world objects of our experience, as when we employ mathematics to calculate some practical result.

The essence of mathematics is its *coherent quality*, a quality found elsewhere, to be sure, but preeminently here. Knowledge of one part of a logical structure entails consequences which are inescapable, and can be found out by reason alone. It is the ability to *deduce consequences* which otherwise would require tedious observation and disconnected experiences to discover, that makes mathematics so valuable in practice; only a confident command of the method by which such deductions are made can bring one the benefit of more than its most trivial results.

Should this coherence of mathematics be inculcated in the schools, at the level K-12, or should it be confined to professional study in the universities? A recent report (17 June 1997) of a task force formed by the Mathematical Association of America to advise the NCTM in its current revisions of the 1989 Standards argues for its early teaching:

> One of the most important goals of mathematics courses is to teach students logical reasoning. This is a fundamental skill, not just a mathematical one . . .
>
> [Teachers] should recognize its theoretical nature, which idealizes every situation, as well as the utilitarian interpretations of the abstract concepts . . .
>
> It should be recognized that the foundation of mathematics is reasoning. While science verifies through observation, mathematics verifies through logical reasoning. Thus the essence of mathematics lies in proofs, and the distinction among illustrations, conjectures and proofs should be emphasized . . .
>
> If reasoning ability is not developed in the students, then mathematics simply becomes a matter of following a set of procedures and mimicking examples without thought as to why they make sense.

(This task force, which continues its work in 1998, is headed by Kenneth Ross, former President of the Mathematical Association of America.)

People untaught in mathematical reasoning are not being saved from something difficult; they are, rather, being deprived of something easy.

Even a small child should understand how the memorization of tables of addition and multiplication for the small numbers (one through 10) necessarily produces all other information on sums and products of numbers of any size whatever, once the structural features of the decimal system of notation are fathomed and applied. At a more advanced level, the knowledge of a handful of facts of Euclidean geometry—the famous Axioms and Postulates of Euclid, or an equivalent system—necessarily imply (for example) the useful Pythagorean Theorem, the trigonometric Law of Cosines, and a veritable tower of truths beyond.

Any program of mathematics teaching that slights these interconnections doesn't just deprive the student of the beauty of the subject, or his appreciation of its philosophic import in the universal culture of humanity, but even at the practical level it burdens that child with the apparent need for memorizing large numbers of disconnected facts, where reason would have smoothed his path and lightened his burden.

People untaught in mathematical reasoning are not being saved from something difficult; they are, rather, being deprived of something easy.

Therefore, in judging standards documents for school mathematics, we look to the "topics" as listed in the "content" criteria not only for their sufficiency at this and that grade level, and not only for the clarity and relevance of their presentation, but also for whether their statement includes or implies that they are to be taught with the explicit inclusion of information on their standing within the overall structures of mathematical reason.

A state's standards will not rank higher by the Reason criterion just by containing a thread named "reasoning," "interconnections," or the like, though what we seek might possibly be found there. It is, in fact, unfortunate that so many of the standards documents we examined contain a

thread called "Problem-solving and Mathematical Reasoning," since that category often slights the reasoning in favor of the "problem-solving," or implies that they are essentially the same thing. Mathematical reasoning is not found in the connection between mathematics and the "real world," but in the logical interconnections within mathematics itself.

Since children cannot be taught from the beginning "how to prove things" in general, they must begin with experience and facts until, with time, the interconnections of facts manifest themselves and become a subject of discussion, with a vocabulary appropriate to the level. Children then must learn how to prove certain particular things, memorable things, both as examples for reasoning and for the results obtained. The quadratic formula, the volume of a prism, and why the angles of a triangle add to a straight angle, for example. What does the distributive law have to do with "long multiplication"; why do independent events have probabilities that combine multiplicatively? Why is the product of two numbers equal to the product of their negatives?

(At a more advanced level, the reasoning process they have become familiar with can itself become an object of contemplation; but except for the vocabulary and ideas suitable and necessary for daily mathematical use, the study of formal logic and set theory are not for K-12 classrooms.)

We therefore shall be looking at the *standards documents as a whole* to determine how well the outlined subject matter is presented in an order, or a wording, or a context, that can only be satisfied by including due attention to this most essential feature of all mathematics.

Mathematical reasoning is not found in the connection between mathematics and the "real world," but in the logical interconnections within mathematics itself.

IV. Negative Qualities, the fourth group, looks for the presence of unfortunate features of the document that injure its intent or alienate the reader to no good purpose. Or, if taken seriously, will tend to cause that reader to deviate from what otherwise good, clear advice the document contains. We shall call one form of it False Doctrine, a phrase that almost explains itself, but will need some examples. The second form will be called Inflation because it offends the reader with fruitless verbiage, conveying no useful information.

Under False Doctrine, which can be either curricular or pedagogical, is whatever text contained in the standards we judge to be injurious to the correct transmission of mathematical information. As with our criteria concerning Content, our judgments can only be our own, as there are disagreements among schools of experts on some of these matters. Indeed, our choice of the name "false doctrine" for this category of our study is a half-humorous reference to its theological origins, where it is a synonym for heresy. Mathematics education has no official heresies, of course; yet if one must make a judgment about whether a teaching ("doctrine") is to be honored or graded zero, *as we are required to do in the present study*, deciding whether an expressed doctrine is true or false is a mere necessity.

The NCTM, for example, officially prescribes the early use of calculators with an enthusiasm the authors of this report deplore, and the NCTM discourages the memorization of certain elementary processes, such as "long division" of decimally expressed real numbers, and the paper-and-pencil arithmetic of all fractions, that we think essential, that should be second-nature *before* the calculator is invoked for practical uses. We must assure the reader that, while we differ with the NCTM on these and other matters, our own view is not merely idiosyncratic, but also has standing in the world of mathematics education, as can even be seen in some of the documents under review. And even were that not true, it is still our duty to make our own stand and to make it clear. We cannot simultaneously credit two opposing positions on what should be learned.

While in general we expect standards to leave pedagogical decisions to the teachers (most standards documents do so, in fact), so that pedagogy is not ordinarily something we are rating in the present study, there are still in some cases standards containing pedagogical advice that we believe undermines what the document otherwise recommends. Advice against memorization of certain algorithms, or a pedagogical standard mandating the use of calculators to a degree we consider mistaken, might appear under a pedagogical rubric in a standards document under consideration. Here is one of the places where our general rule not to judge pedagogical advice fails, for if the pedagogical part of the document gives advice making it impossible for the curricular part, as expressed there, to be accomplished properly, we must take note of the contradiction under this rubric of False Doctrine.

Two other false doctrines are excessive emphases on "real-world problems" as the main legitimating motive of mathematics instruction, and the equally fashionable notion that a mathematical question may have a multitude of different valid answers. Excessive emphasis on the "real-world" leads to tedious exercises in measuring playgrounds

and taking census data, under headings like "Geometry" and "Statistics," in place of teaching mathematics. The idea that a mathematical question may have various answers derives from confusing a practical problem (whether to spend tax dollars on a recycling plant instead of a highway) with a mathematical question whose solution might form part of such an investigation. As the first (January 21, 1997) *Report of the MAA Task Force on the NCTM Standards* has noted,

> It should also be recognized that results in mathematics follow from hypotheses, which may be implicit or explicit. Although there may be many routes to a solution, based on the hypotheses, there is but one correct answer in mathematics. It may have many components, or it may be nonexistent if the assumptions are inconsistent, but the answer does not change unless the hypotheses change.

Again, *constructivism,* a theoretical stance common today, has led many states to advise exercises in having children "discover" mathematical facts, or algorithms, or "strategies." Such a mode of teaching has its values, in causing students better to internalize what they have thereby learned; but wholesale application of this point of view can lead to such absurdities as classroom exercises in "discovering" what are really conventions and definitions, things that *cannot* be discovered by reason and discussion, but are arbitrary and must merely be learned.

Students are also sometimes urged to discover truths that took humanity many centuries to elucidate, the Pythagorean theorem, for example. Such "discoveries" are impossible in school, of course. Teachers so instructed will necessarily waste time, and end by conveying a mistaken impression of the standing of the information they must surreptitiously feed their students if the lesson is to come to closure. And often it all remains open-ended, confusing the lesson itself. Any doctrine tending to say that telling things to students robs them of the delight of discovery must be carefully hedged about with pedagogical information if it is not to be false doctrine, and unfortunately such doctrine is so easily and so often given injudiciously and taken injuriously that we deplore even its mention.

Students are sometimes urged to discover truths that took humanity many centuries to elucidate.

Finally, under False Doctrine must be listed the occurrence of plain mathematical error. Sad to say, several of the standards documents contain mathematical misstatements that are not mere misprints or the consequence of momentary inattention, but betray genuine ignorance.

Under the other negative rubric, Inflation, we speak more of prose than content. Evidence of mathematical ignorance on the part of the authors is a negative feature, whether or not the document shows the effect of this ignorance in its actual prescriptions, or contains outright mathematical error. Repetitiousness, bureaucratic jargon, or other evils of prose style that might cause potential readers to stop reading or paying attention, can render the document less effective than it should be, even if its clarity is not literally affected. Irrelevancies, such as the smuggling in of political or trendy social doctrines, can injure the value of a standards document by distracting the reader, again even if they do not otherwise change what it essentially prescribes.

The most common symptom of irrelevancy, or evidence of ignorance or inattention, is bloated prose, the making of pretentious though empty pronouncements, conventional pieties without content. Bad writing in this sense is a very notable defect, though not the greatest, in the collection of standards we have studied. Some examples will appear below.

We thus distinguish two essentially different failures subsumed by this description of pitfalls, two Negative Qualities that might injure a standards document in ways not classifiable under the headings of Clarity and Content: Inflation (in the writing), which is impossible to make use of; and False Doctrine, which can be used but shouldn't. How numerical scores will be assigned under these headings will be explained in Section VII below; here we signal only their titles, as used in the Table of Ratings there:

IV-A False Doctrine
IV-B Inflation

VI. EXAMPLES FOR THE CRITERIA

Note: The examples given in this section are taken from state standards as named, but it must not be imagined that these are the only states, or the only quotations, that could have been used to the same purpose. Each example, for good or bad, can be matched by many others, from many other states.

Criterion I: Clarity

I-A. A standard should be *clear:*

> Demonstrate understanding of the complex number system (Arkansas, "Number Sense" strand, grades 9-12);

rather than *unclear:*

> Connect conceptual and procedural understandings among different mathematical content areas (Washington, Standard 5.1, Benchmark 2 - grade 7, p.65).

"Conceptual understandings" and "procedural understandings" might mean something, likewise "different . . . content areas," but "connecting" the two former "among" the latter is just too hard to understand, or make use of as a guide to classroom activity.

I-B. A standard should be *definite:*

> Given a pattern of numbers, predict the next two numbers in the sequence (Alaska, Framework, "Reasoning," Level 1: ages 8-10);

rather than *indefinite:*

> Apply principles, concepts, and strategies from various strands of mathematics to solve problems that originate within the discipline of mathematics or in the real world (Alaska Framework, "Problem-solving," Level 3: ages 16-18).

The first example is extremely definite, though as a lesson it might be mathematically questionable. The second ("apply principles . . . to solve problems") is exemplary in its indefiniteness. An *indefinite* standard describes something a teacher has no way of completing ("bringing to closure" is a common phrase for the process of completing a lesson), even if it is quite easy to know if that particular lesson is relevant to the standard.

The wording in this example also illustrates a common failing in definiteness shared by many states at all levels: the use of the word "problem" without a hint of the nature of the problem. Mathematics is preeminently the science of problem-solving, but the "problem" of adding twelve and seven (grade 1) is of a different order from the "problem" of determining the dimensions of a rectangle of given perimeter and area. Every 7th grade teacher in a certain state might understand exactly what is meant by "problem" when the standard for grade 7, under "patterns," uses that word; but this understanding is parochial, an accident of the curriculum and culture traditional in that place and time. Our hypothetical teacher from out-of-state will not know this meaning. When the authors of the present report also cannot discern what is meant, such a standard must be counted inadequate.

The quoted second example from Alaska is actually even worse than this, since it occurs under the rubric "Problem-solving," and ought rather to elucidate that term than assume its meaning is already clear. If the rubric had been "algebra" we would at least know that algebra problems were meant, though this knowledge would still be insufficient to define a standard of accomplishment.

I-C. A standard should be *testable:*

> Analyze spatial relationships using the Cartesian coordinate system in three dimensions (New York, "Four year sequence, Modeling," p.27);

rather than *ineffable* or *not testable:*

> Students [will] utilize mathematical reasoning skills in other disciplines and in their lives (New Jersey Standard 4.4, by the end of grade 8).

It should be plain that the second standard, while desirable as a long-range goal for the teaching of mathematics, is not something one can do more than conjecture about; a teacher has little way of finding out whether it has been accomplished. On the other hand, there are many tests for whether the student can "analyze spatial relationships," whatever interpretation one wishes to put on the phrase. And there are many. The standard concerning "spatial relationships" is not very definite, or clear, but whatever it is, it is testable.

Even though "clear," "definite," and "testable" are not the same thing, they are a family of qualities that tend to go together. Standards that fail any of these three tests announce themselves to the reader as somehow unclear,

albeit in different ways. We therefore group them together, and can think of no better overall title for the grouping than Clarity.

The idea here is that our imagined teacher in a state offering standards highly rated by us for each entry under Clarity should be in no doubt about what to explore, explain and examine, how to judge whether a book covers the ground described, or what to test the children upon at each stage in their progress. A poor score, on the other hand, signals a standards document from which he can glean little or no help at all on what the state expects of him in his daily tasks, and will have to rely on his own insight and experience to guess at what it might be, even though the intended content of the state curriculum might be adequate and clear in the mind of the state's educational advisors.

Criterion II: Content

Content will be rated separately for the categories Primary, Middle, and Secondary, in most cases meaning K-6, 7-9, and 10-12, with a year's leeway at either end of each segment, according to the way each state divides its content standards. Our grades will observe the state's own divisions, and therefore will be applicable to slightly different segments of the K-12 program for different states. To decide whether a state is asking its children to learn the right things at the right times, and enough such things, and not an unreasonable amount either, the judge must have a set of standards of his own. The authors of this report do indeed have them, though for purposes of judging others we must be rather flexible, there being more than one way to go about securing a given result by the end of the schooling process.

Much as we would like to print here a complete exemplary curriculum that conforms to our own standards, the task would be prohibitively long (as every state well appreciates that has appointed a committee to do just that) and the resulting document would overwhelm the present report. Nor is it possible for a few sentences from a sample standards document to illustrate sufficiency, which must be judged by its entire extent. Hence, we shall not offer any quotations to serve as good and bad examples, as we do for criteria groups I, III, and IV. Just the same, since the reader deserves some notion of where we stand on some of the crucial topics that must form part of every mathematical education, it is worth stating a few desiderata briefly:

We believe the traditional arithmetic of fractions and decimals should be complete by grade 6, but with more understanding of logical connections than was customary 50 years ago and with many more applications than the traditional storekeeping and mensuration skills. We wish the middle grades to introduce geometry and algebra and the logic of equations and their application, and not be the "review of elementary math, plus ratios" that has been common in recent years. We wish the secondary curriculum to be mathematics as the mathematics profession understands it: not a collection of rules for algebra, trigonometry, graphing and the like, but an organized body of knowledge, albeit mainly of algebra, geometry and the elementary functions, with application to human affairs clearly distinguished from the inner logic of the mathematics itself—and both of them fully represented.

To describe a curriculum in so few words is, of course, insufficient; but we might point to several of the state documents as models. None of them is a model in all respects, but the reader may deduce from the scores (in Part VII, or in the Notes on that State in the following section) which states describe the content we consider sufficient. They are all publicly available (see Appendix). While not the highest overall, Alabama, Arizona, and Tennessee (at the 8-12 level) are high-ranking for content, and California, North Carolina, and Ohio are good models in both content and the other categories. We have also included high scores for Japan, which presents a document much different from any of the American states, but which is exemplary in content when rightly read, and interesting in other regards as well. We shall have more to say about the Japanese standards in the Notes on the States (Part VIII). There is no single scale for content; what is left out of one listing might simply have been omitted in order that something else might be included, so that states with equal scores will not have identical intentions concerning content. This, too, is a reason for not trying to include a model curriculum description here.

Criterion III: Reason

There is no single place, grade level, or strand, where one can find whether a state standards document exhibits the guidance being graded under this criterion. The mere inclusion of a category of instruction labeled "Mathematical Reasoning" or the like is no guarantee that its contents serve the purpose. To the contrary, evidence of the demand for reasoning is more often found elsewhere, if at all, inextricably bound up with the mathematical content. Quoted examples can only indicate in part the tone of the document as a whole, and it is the whole which gives rise to our rating. At the risk of being unfair to the two documents *as a whole* we shall nonetheless give a pair of such examples, one exhibiting a poor incorporation of the ideal of fostering mathematical reasoning, and the

other a good and clear indication of the kind of lesson that is designed to teach such reasoning and give the student an appreciation of the coherence of mathematics.

Vermont, which has a thread named "Mathematical Problem Solving and Reasoning," lists 17 specifics, some under the grade 9-12 column, some under the grades 5-8 column, and some under the grades K-4 column. Here are six of them (p. 7.4):

> Students [shall]
> (a) Solve problems by reasoning mathematically with concepts and skills expected in these grades.
> (b) Create and use a variety of approaches, and understand and evaluate the approaches that others use; determine how to break down a complex problem into simpler parts; extract pertinent information from situations.
> (c) Formulate and solve a variety of meaningful problems.
> (d) Formulate and solve meaningful problems in many kinds of situations using grade-related mathematical concepts and reasoning strategies.
> (e) Extend concepts and generalize results to other situations.
> (f) Work to extend specific results and generalize from them.

That these instructions are vague and uninformative is evidenced by the fact that the reader would be hard put to decide which grade level, K-4, 5-8, or 9-12, any of these six specifications is intended to apply to. It is therefore clear that there is no *progression* of logical skill, or problem-solving skill, or theoretical understanding, to be deduced from these six statements. In fact, (a) and (e) are listed under K-4; (b) and (c) under 5-8, and (d) and (f) under 9-12. Notice that (a) and (d) are nearly identical, yet (a) is for K-4 and (d) is for 9-12. The main difference appears to be that in 9-12 the "problems" are to be *meaningful*, an ill-defined idea having no relevance to either problem-solving or reason.

Indeed, any of the six instructions can be construed to apply to any levels whatever, including the editors of a mathematical research journal such as the *Proceedings of the American Mathematical Society.* And the editor of even that journal might hesitate before grandly asking an author to "extend concepts and generalize results to other situations," as was here prescribed for children at the level K-4.

Surely the Vermont authors had something more specific in mind, but it does not come across in the framework they actually wrote. Some demands for mathematical reasoning do occur in other parts of Vermont's *Framework,* to be sure, notably in the section headed "Mathematical Understanding," where certain required skills are mentioned; and all of this must be taken into account in giving an overall score for Reason. A high score would require that the intended lessons in reasoning, or making connections, be *part of* a sufficient number of the specific content demands throughout the document as to make plain what lessons in reasoning are intended; but Vermont's section, quoted (in part) here, that includes the word "reasoning" fails to do so. It contains exhortations, not standards.

On the other hand, Virginia does show the proper quality clearly enough in some places to make it convenient to quote a selection by way of contrast:

> **(Grade 2)** The student, given a simple addition or subtraction fact, will recognize and describe the related facts which represent and describe the inverse relationship between addition and subtraction (e.g., 3 +__ = 7, ... , 7 - 3 = __).
>
> **(Grade 5)** Variables, expressions and open sentences will be introduced. . . .
>
> **(Grade 6)** The student will construct the perpendicular bisector of a line segment and an angle bisector, using a compass and straight-edge.
>
> **(Geometry)** The student will construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include . . . identifying the converse, inverse and contrapositive of a conditional statement. . . .
>
> **(Algebra II)** The student will investigate and describe the relationships between the solution of an equation, zero of a function, x-intercept of a graph, and factors of a polynomial expression. . . .

These examples from Virginia's standards are, with one exception concerning pure logic, simple instructions as to *content,* but presented in a manner that carries the proper message of the unity of mathematics, and the reasoning by which its parts are held together. Five such examples do not, of course, amount to a curriculum in mathematical reasoning, and in fact Virginia is, like most states, somewhat lacking in these qualities overall; but these five should serve as illustration of the standard desired; whereas mere exhortation, to "make connections," "use . . . a variety of reasoning strategies," and the like, are not sufficient guidance, and simply are not usable in the absence of a referent.

An instruction of the sort that should appear more often is also found in The District of Columbia's (Mathematics)

Curriculum Framework, where on page M-5, under "What a student should know and be able to do by the end of Grade 11," appears the following:

> 5. Explain the logic of algebraic procedures.

This is not completely stated, and so its intent must be inferred; but most of the documents under review don't even go this far in reminding the reader that the logic of a procedure is essential to its intelligent use.

A more complete statement of the same desideratum may also be found in Pennsylvania's standards draft for the 6th grade level "Algebra and Functions" strand. Students here are to:

> 1. write verbal expressions and sentences as algebraic expressions and equations, and solve them, graph them and interpret the results in all three representations.
>
> 1.1 . . . write and solve one-step linear equations in one variable.
>
> 1.2 . . . write and evaluate an algebraic expression for a given situation using up to three variables.
>
> 1.3 . . . apply algebraic order of operations and the commutative, associative and distributive properties to evaluate expressions and justify each step in the process.

Again, Pennsylvania's proposed "Standards" offers, at the grade 11 level,

> Prove two triangles or two polygons are congruent or similar using algebraic and coordinate as well as deductive proofs.

The Pennsylvania instruction is a little vague, especially for "polygons," in that it does not specify the sort of hypotheses that are likely to have been in place when the proof was to be done; but the idea is plain, the lesson valuable, and the word "prove" a delight to encounter.

Criterion IV: Negative Qualities

IV-A False Doctrine

A standard must not offer advice which, if followed, will subvert instruction in the material otherwise demanded:

First Example

For 7th grade, under Standard: 5070-07 ("The students will develop number concepts underlying computation and estimation in various contexts by performing the operations on a pair or set of numbers."), Utah asks as #3 of "Skills and Strategies,"

> Develop an algorithm for multiplication of common fractions and mixed numbers by using models or illustrations; explain your reasoning.

This suggestion is misleading in that "develop" implies that the conventions concerning the multiplication of fractions are somehow to be *discovered* by the students, and indeed then *justified* by them. In fact the very definition of multiplication for fractions is conventional, not natural and "discoverable." It is a triumph of the development of mathematics that the rule for the multiplication of fractions is both a consistent extension of the multiplication and division in the ring of integers, and at the same time interpretable as an operation concerning measurements in geometry and partitioning in finite sets. That the definitions and results are independent of the "fraction" representation of the rational numbers involved, i.e., of the denominator chosen, is another complication. As with any other convention or fact, this cannot be *discovered,* though its invention and its rationale should be thoroughly taught.

That 7th-grade students should *participate* in discussions of these matters is of course to be desired, but a teacher who feels obliged to set up a classroom environment in which students "develop an algorithm . . ." will be wasting time, and fooling either himself or his students; while possibly neglecting the truly interesting features of the system of rational numbers and its possible uses.

Second Example

Arizona, in "Patterns, Algebra, and Functions," grades 9-12, asks students to analyze the effects of parameter changes in formulas defining functions, "using calculators."

This curriculum obviously intends the student to understand the effects of parameter changes, but believes the use of (graphing) calculators sufficient for the purpose. It is not sufficient for the purpose. Attention to the symbolic expressions, individual point calculations and point plotting, are a necessity. True, calculators are a valuable aid, even a necessity, in serious use with complicated data in science and in business; but that is life, not instruction, and the proper use of calculators must follow, not precede, the logical, conceptual understanding of the effects of parameter changes. A standard that instructs teachers to confine themselves to calculators in exercises, where they will not suffice for instructional purposes, is destructive of the lesson intended in that part of the curriculum. Omitting the phrase "using calculators" might have made this standard a good one, for the desideratum (to under-

stand the effect of parameter changes) is clear and valuable; but the method of getting there should rather have been left to the teacher. There doubtless is a place for calculators somewhere in there, but this Arizona pedagogical instruction preempts the choice in an unfavorable direction.

The widespread prescribing of calculators in the early grades is also destructive of learning, for the algorithms concerning fractions and decimal multiplication and division are not mere "19th Century skills" now rendered obsolete by technology, but when properly taught inculcate "number sense" and the ability to estimate; and serve as essential preparation for the more sophisticated operations of later years, such as the algebra of rational functions.

Third Example

Alabama, in its general introduction to the detailed list of standards that follows, has a section headed "Use of Manipulatives," which contains this directive:

> Throughout their schooling, students should be involved in activities in which manipulatives are used to aid in conceptual and procedural understanding. . . . Use of manipulatives helps . . . clarify algorithms . . . Using manipulatives also richly illustrates the connection between concrete experiences and abstract mathematics.

In the early childhood years, the use of manipulatives, such as blocks, Cuisinaire rods, and surely the abacus, can give a child the basis in experience that must underlie the abstractions of mathematics; and the world has used them since the dawn of mankind. But "throughout" their schooling is an error, and has moreover led to an unnecessarily large manipulatives industry allied to textbook publishing, whose products decorate the displays at teachers' conferences and professional journals.

Mathematics is not raw experience; it is an analogue for experience. As it becomes more sophisticated, as with algebraic equations representing physical relationships, it must necessarily supplant that which can be manipulated; otherwise, the lesson of algebra is lost. A teacher who takes seriously the Alabama instruction as given will find himself looking for ways to employ such curiosities as "algebra tiles" in teaching the solution of equations, and literal balances for the understanding of just those methods that were designed to render literal balances unnecessary. Rather than look for ways to use manipulatives, teachers should strive for ways to wean students from their use, little by little, as a potential bicyclist must be weaned from training wheels.

IV-B Inflation

A standards document must not employ language whose purpose can only be to fill paper, nor must it suggest a profundity impossible for the level in question, especially if the indications are that the author doesn't understand the words being used. Such language weakens the respect the user of the document should have for valid parts of the document, even where it does not result in outright error or false doctrine.

Bad writing in this sense differs from vagueness or other failures of clarity in that it is not simply mistaken, incomplete or obscure, but in that it is pretentious and cannot be taken seriously, or is empty of content altogether. If it does include a definite instruction, that instruction is generally impossible of execution, at least at the level of instruction in question.

Example 1

In Michigan's *Model Content Standards for Curriculum (Mathematics)*, Standard 8 states:

> Students draw defensible inferences about unknown outcomes, make predictions, and identify the degree of confidence they have in their predictions.

Then the first item under this standard, identically worded and repeated at each of the levels primary, middle school, and high school, is

> Make and test hypotheses.

As these four words constitute a short description of the entire science of statistics (and indeed of all science), this "standard" is impossibly broad, hence useless. Later items indicate certain particulars, e.g., "Make predictions and decisions based on data, including interpolations and extrapolations." This is broad enough, but does include some clues (e.g., "interpolations") limiting the intended lessons, though even here much the same wording is used at all three levels. This particular statement can be called, generously, "too vague," but "Make and test hypotheses" is worse, and is no guide to either curriculum or instruction.

Example 2

Oklahoma's "Priority Academic Student Skills," at the grade 2 level, under the rubric "Mathematics as Connections" requires that the student will

A. Develop the link of conceptual ideas to abstract procedures.
B. Relate various concrete and pictorial models of concepts and procedures to one another.

C. Recognize relationships among different topics in mathematics.
D. Use mathematics in other curriculum areas.
E. Use mathematics in daily life.

Now, B, D, and E are understandable, even though they do not say anything very specific. Children in grade 2 typically learn the names and notations for integers up to a thousand, and how to add and subtract some of them, perhaps making change at a grocery store; they learn some geometric language and so on. It is hard to see why their curriculum should be subjected at this level to a rubric such as "Mathematics as Connections" at all, except that there are theorists who insist that all categories by which mathematics may be classified should obtain at all levels. Condemned to follow out this theory for grade 2, the authors could hardly avoid having to invent empty items such as A and C to inflate the list.

Example 3.

Hawaii's *Essential Contents* ("Geometry and Spatial Sense," Grades 9-12) claims to prescribe

> Non-Euclidean geometries; and
> Hyperbolic and elliptical geometries.

This shocking entry perhaps springs from its authors' having seen a popularization of these topics somewhere and deciding it to be a pleasant sort of thing to talk about in school. But, like capsule biographies of famous mathematicians and magazine articles about Fermat's Last Theorem, popularizations of arcane mathematical theories are not curriculum. A serious lesson in non-Euclidean geometries is impossible at this stage of schooling. Even Euclidean geometry is slighted in Hawaii's *Essential Contents*, and non-Euclidean geometry can no more be understood in the absence of Euclid's structure than night can be understood in the absence of day.

(There is a possibility, evidenced in some advertising we have seen, that the [Euclidean] properties of the sphere are regarded as "non-Euclidean geometry" by some manufacturers of manipulatives to be used in the schools. If that is the origin of the mention of "non-Euclidean geometry" in several of the state standards, it is a poor use of the phrase, and as a lesson in geometry is inferior to the deductive Euclidean geometry it appears to replace.)

Again, under Hawaii's rubric "Patterns and Relationships" we find that

> Students develop algebraic thinking through . . . [among other things] . . . Topological concepts.

In a curriculum guide that nowhere mentions an axiom, theorem, or deductive argument, this is also unrealistic, even apart from the problematic association of "algebraic thinking" at the high school level with such "topological concepts" as might possibly be found there. Are these "Essential Content" items for real? No. They are Inflation.

VII RATING THE STATES: A TABLE OF RESULTS

Every state is listed in the following table for the sake of completeness, though several of them are ungraded, marked "n" rather than with one of the numbers 0, 1, 2, 3, or 4, because (except for the special case of Tennessee) they do not have a standards document at all (Iowa) or because they have standards in process of revision, with no draft publicly available for comment (Minnesota, Nevada, and Wyoming). The State-by-State Notes (Section VIII) that follow the table of grades will explain the case of Tennessee, which has two sets of scores but with a single final (average) grade for its two documents.

The collapse of deductive reasoning as a desideratum in American school mathematics is the single most discouraging feature of the study of these documents.

Giving grades to states differs somewhat from giving grades to students in school or college. In grading students, teachers are dealing with youngsters taking several courses, not all of which interest them greatly. The students are pressed for time on examinations, and during the week as well, and are sometimes fearful, ill, or troubled by personal problems. We wish to encourage students to improve, or reward them for performance we know about but which is not present in the written work they present. In consequence, we give many grades of "A" for less than exemplary performance. The world rather expects that 10% of a class (or 5%, or 15%) will receive grades of A (i.e., 4 grade points).

A state publishing standards, however, is not an anxious youngster pressed for time. A state has time and money and the ability to secure expert advice. It can consult books and other states. It should be without psychological problems or learning disabilities. This particular "assignment," the writing of standards, is a professional responsibility, and any performance short of exemplary must be judged so, and announced so, even if the top grade were received by none of them.

Thus, while we have used the scale 4, 3, 2, 1, 0 in more or less the usual way, "4" representing the best available and "0" representing the least useful, these figures must be seen simply as a ranking of value, five degrees of excellence being about as many as we can distinguish. It was only after tabulating all the numerical scores that we decided which total scores should accord with the more familiar scheme of A, B, C, D, F—grades which are given as the right hand column of the chart.

The grading for Negative Qualities might seem a bit curious, grades of 4 being awarded for the *absence* of False Doctrine, or of Inflation, and 0 for those states having the most; but, since a total score was needed for computing the final grade of A, B, C, D, or F, it seemed convenient to scale negations positively, in order to be able to use additions only to arrive at a total. Otherwise some states would have ended with negative scores. However, this method of scoring negative qualities would give a state publishing a standards document containing no words at all a grade of 4 points, because of its perfection in achieving the absence of inflation and false doctrine; and as it turned out this would have been just sufficient to earn a D. Thus, the 16 States that received a final evaluation of "F" would have scored better, or as well, in our final tally if they had turned in a blank paper. (With negative scores in part of the document, it seems, as in Minkowskian geometry, there is simply no way to avoid at least some anomaly.)

General Impressions

The collapse of deductive reasoning as a desideratum in American school mathematics is the single most discouraging feature of the study of these documents. The second, strikingly evidenced by the paucity of grades of 4 at the Primary level for Content (Criterion II), and for False Doctrine (Criterion IV(B)), was the enthusiasm with which many states have embraced the recent doctrine that the algorithms for multiplication and division of fractions and decimals are obsolete and can be replaced by calculators. Indeed they are, for technicians who need the numerical answers for practical everyday purposes; but instructional purposes demand otherwise. We did not demerit the prescribing of technology *per se*, at the primary or any other level, but we did downgrade its prescription in places where its use obscures a *mathematical* lesson, or blunts a *mathematical* perception, that can only be conveyed in its absence. And there are many such places, from the decimal calculations of the 4th grade to the solution of linear systems in the 12th. Most failures found in these standards documents require more detail of presentation, and an effort is made in the Notes (Section VIII) to describe a representative selection of them.

In addition to all the states we have named below, one other Standards is graded for comparative purposes: Japan, which appears at the end of the listing. The Japanese standards document is a translation of a publication of the Japanese Ministry of Education, and is listed in the Appendix, with information on its provenance. This

document, while brief, is exemplary in most respects. Even so, it falls short of our maximum in the category of Reason (III), perhaps only due to deficiencies in the translation, or to a cultural difference that makes it seem unnecessary to the Japanese to mention the matter sufficiently often when describing content. We must judge only by what we can read, however. (See also the Note on Japan in Section VIII below.)

There are nine scores (of 0 to 4) for each state, but for evaluation purposes there are but four Categories, I, II, III, and IV (as described above), for each of which an average is struck before the four averages are added for a total score for the state. That is, we are weighting equally each of the criteria—Clarity, Content, Reason, and Negative Qualities—even though some are split into more subheads than others. Thus, 16 is the highest possible total score.

TABLE 1. NUMERICAL RATINGS FOR THE STATES

STATE	CLARITY			CONTENT			REASON	NEG. QUALITIES		TOTAL	GRADE
	Language I (A)	Reference I (B)	Testability I (C)	Elem. II (A)	Middle II (B)	Second. II (C)	Reasoning III	Abs. of False Doctrine IV (A)	Abs. of Inflation IV (B)		
Alabama	3	3	3	4	4	4	1	3	4		
AL (avg)		3.0			4.0		1.0	3.5		11.5	B
Alaska	1	0	1	2	4	1	2	3	2		
AK (avg)		0.7			2.3		2.0	2.5		7.5	C
Arizona	2	3	3	4	4	4	2	4	3		
AZ (avg)		2.7			4.0		2.0	3.5		12.2	B
Arkansas	0	0	0	0	0	0	1	3	0		
AR (avg)		0.0			0.0		1.0	1.5		2.5	F
California	4	4	4	4	4	4	4	4	4		
CA (avg)		4.0			4.0		4.0	4.0		16.0	A
Colorado	0	1	1	1	2	2	1	2	2		
CO (avg)		0.7			1.7		1.0	2.0		5.4	D
Connecticut	0	1	1	1	1	1	1	2	2		
CT (avg)		0.7			1.0		1.0	2.0		4.7	D
Delaware	1	1	1	4	1	2	2	3	2		
DE (avg)		1.0			2.3		2.0	2.5		7.8	C
District of Columbia	1	0	1	0	0	0	1	3	3		
DC (avg)		0.7			0.0		1.0	3.0		4.7	D
Florida	2	1	1	2	2	2	0	1	2		
FL (avg)		1.3			2.0		0.0	1.5		4.8	D
Georgia	4	3	4	3	3	4	1	3	4		
GA (avg)		3.7			3.3		1.0	3.5		11.5	B
Hawaii	0	0	0	0	1	0	0	2	0		
HI (avg)		0.0			0.3		0.0	1.0		1.3	F
Idaho	1	1	0	1	2	0	0	0	1		
ID (avg)		0.7			1.0		0.0	0.5		2.2	F
Illinois	2	0	2	2	2	2	1	2	3		
IL (avg)		1.3			2.0		1.0	2.5		6.8	D
Indiana	4	3	3	3	2	1	0	2	4		
IN (avg)		3.3			2.0		0.0	3.0		8.3	C
Iowa	n	n	n	n	n	n	n	n	n		N
Kansas	2	2	1	2	2	0	0	0	3		
KS (avg)		1.7			1.3		0.0	1.5		4.5	D
Kentucky	1	1	1	1	2	2	0	3	2		
KY (avg)		1.0			1.7		0.0	2.5		5.2	D
Louisiana	1	0	0	0	0	0	0	0	2		
LA (avg)		0.3			0.0		0.0	1.0		1.3	F
Maine	0	0	0	0	2	1	0	3	2		
ME (avg)		0.0			1.0		0.0	2.5		3.5	F
Maryland	0	1	1	2	2	0	0	2	0		
MD (avg)		0.7			1.3		0.0	1.0		3.0	F
Massachusetts	1	1	1	0	1	0	0	0	0		
MA (avg)		1.0			0.3		0.0	0.0		1.3	F
Michigan	0	0	1	1	1	1	0	0	0		
MI (avg)		0.3			1.0		0.0	0.0		1.3	F
Minnesota	n	n	n	n	n	n	n	n	n		N
Mississippi	4	3	4	3	4	4	2	3	2		
MS (avg)		3.7			3.7		2.0	2.5		11.9	B
Missouri	0	1	1	2	2	2	0	0	0		
MO (avg)		0.7			2.0		0.0	0.0		2.7	F
Montana	0	0	0	1	0	2	0	1	0		
MT(avg)		0.0			1.0		0.0	0.5		1.5	F

TABLE 1. NUMERICAL RATINGS FOR THE STATES (continued)

STATE	CLARITY			CONTENT			REASON	NEG. QUALITIES		TOTAL	GRADE
	Language I (A)	Reference I (B)	Testability I (C)	Elem. II (A)	Middle II (B)	Second. II (C)	Reasoning III	Abs. of False Doctrine IV (A)	Abs. of Inflation IV (B)		
Nebraska	0	0	0	0	1	2	1	0	2		
NE (avg)		0.0			1.0		1.0	1.0		3.0	F
Nevada	n	n	n	n	n	n	n	n	n		N
New Hampshire	2	2	1	2	2	2	1	2	3		
NH (avg)		1.7			2.0		1.0	2.5		7.2	C
New Jersey	2	1	3	3	4	1	2	2	3		
NJ (avg)		2.0			2.7		2.0	2.5		9.2	C
New Mexico	0	0	0	1	1	1	0	3	0		
NM (avg)		0.0			1.0		0.0	1.5		2.5	F
New York	1	1	2	2	3	4	3	4	4		
NY (avg)		1.3			3.0		3.0	4.0		11.3	B
North Carolina	4	4	4	3	4	4	3	3	4		
NC (avg)		4.0			3.7		3.0	3.5		14.2	A
North Dakota	2	1	1	3	3	1	0	2	2		
ND (avg)		1.3			2.3		0.0	2.0		5.6	D
Ohio	4	2	3	4	4	4	3	3	4		
OH (avg)		3.0			4.0		3.0	3.5		13.5	A
Oklahoma	1	0	1	2	2	1	0	3	0		
OK (avg)		0.7			1.7		0.0	1.5		3.9	F
Oregon	2	1	1	1	1	1	1	2	2		
OR (avg)		1.3			1.0		1.0	2.0		5.3	D
Pennsylvania	2	1	2	3	2	2	0	1	2		
PA (avg)		1.7			2.3		0.0	1.5		5.5	D
Rhode Island	0	0	0	1	2	0	0	0	0		
RI (avg)		0.0			1.0		0.0	0.0		1.0	F
South Carolina	2	1	1	3	3	2	0	0	1		
SC (avg)		1.3			2.7		0.0	0.5		4.5	D
South Dakota	0	0	0	0	0	0	0	3	0		
SD (avg)		0.0			0.0		0.0	1.5		1.5	F
Tennessee (K-8)	0	0	0	2	1	n	0	2	0		
TN (K–8) (avg)		0.0			1.5		0.0	1.0		2.5	F*
Tennessee (9-12)	4	4	4	n	n	4	3	3	3		
TN (9–12) (avg)		4.0			4.0		3.0	3.0		14.0	A*
*Partial grades; Tennessee Average for both documents										8.25	C
Texas	4	3	3	3	3	3	1	4	3		
TX (avg)		3.1			3.0		1.0	3.5		10.6	B
Utah	4	2	2	3	3	3	3	3	3		
UT (avg)		2.7			3.0		3.0	3.0		11.7	B
Vermont	2	1	1	4	4	1	1	4	2		
VT (avg)		1.3			3.0		1.0	3.0		8.3	C
Virginia	4	4	4	3	2	2	2	3	4		
VA (avg)		4.0			2.3		2.0	3.5		11.8	B
Washington	0	0	0	1	2	0	0	3	0		
WA (avg)		0.0			1.0		0.0	1.5		2.5	F
West Virginia	4	3	3	3	4	4	2	3	4		
WV (avg)		3.3			3.7		2.0	3.5		12.5	B
Wisconsin	2	1	2	3	4	1	0	3	3		
WI (avg)		1.7			2.7		0.0	3.0		7.4	C
Wyoming	n	n	n	n	n	n	n	n	n		N
Japan	4	4	4	4	4	4	3	4	4		
JN (avg)		4.0			4.0		3.0	4.0		15.0	A

VIII. NOTES, STATE-BY-STATE

The following Notes are not intended as full explanations of the numerical scores tabulated in Part VII above and repeated here state-by-state. Those explanations are implicit in the numbers themselves, coupled with the description of the criteria given earlier in this report. To review all the details that entered the scores would require more space than is possible.

In particular, the quotations are not necessarily drawn from or based on the most typical or characteristic features of the document in question. On the whole, they are selected to provide, *in their ensemble*, a glimpse of some of the detail, here and there, that no summary or averaging procedure can provide. Since these notes mainly (though not entirely) exhibit weaknesses rather than strengths in the documents mentioned, we expect that, taken together, they provide additional insight into what, by our criteria, is the overall failure of the current efforts at writing useful state standards for school mathematics. That certain particular criticisms appear under the headings of particular states does not mean they are peculiar to the State in which they appear. Many are applicable to other states, and the objects of these criticisms tend to occur in much the same form—sometimes in exactly the same form—in the standards documents of other states.

Alabama

The Content is one of the most comprehensive, and is mostly, though not always, direct in language. A typical good item: "Describe characteristics of plane and solid figures using appropriate terms. Examples: round, flat, curved, straight" (1st grade, p. 18). Too often, on the other hand, occurs something like this: "Describe, extend, and create a wide variety of numeric and geometric patterns" (5th grade, p. 46). This is not a "learning objective" at all, and even as a pedagogical device is too broad and open-ended.

There is a lot of content in the course descriptions, though including some old-fashioned items that really should be retired as unilluminating and time-wasting, such as Descartes' Rule of Signs and Synthetic Division. The document as a whole slights deductive reasoning, which is principally mentioned in the geometry courses, but not carefully outlined even there, where its use is most traditional. It is surely evidence of this lack of attention to logical structures that the authors assign this impossible task: "Use the Fundamental Theorem of Algebra to solve polynomial equations" (p. 35) which mistakes the logical status of an existence theorem. Yet the document as a whole is one of the best.

STATE REPORT CARD

Alabama

I. CLARITY	3.0
II. CONTENT	4.0
III. REASON	1.0
IV. NEGATIVE QUALITIES	3.5
TOTAL SCORE (out of 16)	11.5
GRADE	B

Alaska

The *Framework* (1) is extremely vague, offering examples of lack of clarity, and also of inflation: "By strategically applying different types of logics, students will learn to recognize which type of logic is being used in different situations and respond accordingly." ("Different types of logic" is, we believe, a reference to induction and deduction, about which much is made these days, without much effect. And "respond accordingly" couldn't be vaguer.) On page 4-12, under "benchmarks" for Math Content Standard A (Content of Math), at the 16-18 year-old level, the following appears: "A student would be able to . . . explore linear equations, nonlinear equations, inequalities, absolute values, vectors and matrices." "Exploration" is not a standard, and "absolute values" is curiously misplaced in this list. Clarity has suffered here, and more than clarity.

At the grade 3-5 level, under "Problem Solving," readers are told, "Evaluate the role of various criteria in determining the optimal solution to a problem." This is not only unclear, but is Inflation. For high school: "Recognize how mathematics changes in response to changing societal needs." (We believe mathematics is eternal and unchanging. There certainly are many things that vary in response to social pressures, but the document puts it badly.) The *Standards* (2) partly makes up for the deficiencies in the *Framework* (1), especially in its avoidance of inflated language, but it outlines a program lacking in sufficient content, especially at the secondary level.

STATE REPORT CARD

Alaska

I. CLARITY	0.7
II. CONTENT	2.3
III. REASON	2.0
IV. NEGATIVE QUALITIES	2.5
TOTAL SCORE (out of 16)	7.5
GRADE	C

Arizona

The "performance objectives" are very briefly stated, and, while the reader has to make some difficult inferences, they are apparently demanding and good. Also, Arizona lists extra objectives for honor students, a useful feature. Sometimes, however, ambition outruns the language. For example, under "Geometry 4M-P4," "PO 3: State valid conclusions using given definitions, postulates and theorems" (p. 21). This is inflated and indefinite, as none of the surrounding text asks for knowledge or skills that would give it substance. Taken seriously, so vast an instruction should be broken down into numerous graded demands, describing one or more years of algebra or geometry. (Curiously, no particular subject matter is mentioned here.) Thus, Reason is badly outlined as a thread in an otherwise comprehensive document; it is put into one corner of the curriculum, as it were. As for Inflation, here is an example of unreality at the "honors" level in high school: "Demonstrate technical facility with algebraic transformations, including techniques based on the theory of equations." The "theory of equations" is ill-defined today, and as understood 50 years ago concerned things about real polynomials that did not really translate into "techniques" concerning transformations. Curiously enough, this standard is also found *verbatim* in Idaho's "algebra" strand, at the 9-12 level (see below).

STATE REPORT CARD

Arizona

I. CLARITY	2.7
II. CONTENT	4.0
III. REASON	2.0
IV. NEGATIVE QUALITIES	3.5
TOTAL SCORE (out of 16)	12.2
GRADE	B

Arkansas

The telegraphic style of this *Framework* may conceal a good program in many school districts, but it says so little that it cannot be of much use. "Use technology" is a sentence that occurs repeatedly and often pointlessly. Standard 5.2.9, "Use mathematical reasoning to make conjectures and to validate and justify conclusions and generalizations," is no more helpful than to say, "Use mathematics." Standard 5.2.16, "Apply algebraic processes to non-algebraic functions," is opaque. Standard 2.1.6 (grades 9-12), "Explore non-Euclidean geometries," is unreasonable where Euclidean geometry is already too little explored.

> **"Use mathematical reasoning to make conjectures and to validate and justify conclusions and generalizations," is no more helpful than to say, "Use mathematics."**

(In fact, "non-Euclidean geometry" in today's high schools, where it is mentioned at all, sometimes designates spherical geometry, which is quite Euclidean. If this is what is meant, it should be stated; if this is not what is meant, it is inflation, since what the history of mathematics calls "Non-Euclidean Geometry" is any of a number of sophisticated axiomatic systems that differ from the Euclidean system *only* in postulating alternatives to the famous parallel postulate.)

The best feature is the document's relative lack of outright False Doctrine, but even in its brevity there is a great deal of inflation, e.g., "Visualize algebra as a bridge between arithmetic and higher level mathematics" (p. 9). This is neither clear nor definite nor testable, nor yet an item of content nor of reasoning. Yet it is labeled a "learning expectation."

STATE REPORT CARD

Arkansas

I. CLARITY	0.0
II. CONTENT	0.0
III. REASON	1.0
IV. NEGATIVE QUALITIES	1.5
TOTAL SCORE (out of 16)	2.5
GRADE	F

California

The *Standards* is a scant 37 pages in length, and is classified by grade level from K-7 and then by subject headings (the subjects are strangely called "disciplines"), from Algebra I (in Grade 8) through courses which prepare the student for AP Calculus and AP Statistics. The writing is always terse and to the point.

At the start of each grade (K-7), the expectations for that grade are summarized in a hundred words or so, permitting a rapid and accurate overview of the whole. The details follow by rubric, the same for all grade levels:

i Number Sense
ii Algebra and Functions
iii Measurement and Geometry
iv Statistics, Data Analysis, and Probability
v Mathematical Reasoning

Although naming a rubric "Algebra and Functions" is stretching things at the lower levels, there is in general no undue multiplicity of rubrics, e.g. a strand labeled "Calculus" which at least one other state mysteriously included in its framework right down to the first grade. "Mathematical reasoning" is the only one here whose presence might be questioned, for its demands are of a rather general nature, and some could be considered "inflation" by the authors of this report if they were not thoroughly exemplified in the content standards of the other rubrics.

Here the "reform" philosophy of what its opponents sometimes have called "fuzzy math" is firmly rejected.

Each rubric is headed by one or more general admonitions which would also tend to be labeled not "Definite" or "Testable" were it not that their subheadings explain exactly what is meant. Under "Algebra and Functions," grade 3, for example, students are to "...represent simple functional relationships" (which is vague to say the least), but then the provincial teacher imagined in the Criteria is immediately told what that means in terms of content, e.g. "solve simple problems involving a functional relationship between two quantities (e.g., find the total cost of multiple items given the per unit cost)."

Again, under "Mathematical Reasoning," grade 3, we find that "Students use strategies, skills and concepts in finding solutions." So often admonitions of this sort are simply left hanging as empty exhortations, but here follow six specifications, e.g., "express the solution clearly and logically using appropriate mathematical notation and terms and clear language, and support solutions with evidence, in both verbal and symbolic work." A tall order, perhaps, but conveying (in passing) another important point: In speaking of "the" solution, the phrasing insists that mathematical problems have a single solution. Here the "reform" philosophy of what its opponents sometimes have called "fuzzy math" is firmly rejected.

In grade 4, instead of reading "Investigate the relation between the area and the perimeter of a rectangle" (a popular, though confusing, entry in many state standards, and in any case "investigate" is not a content standard), we find (1.2) "recognize that the rectangles having the same area can have different perimeters," and (1.3) "understand that the same number can be the perimeter of different rectangles, each having a different area." It is also refreshing to observe, in grade 4 "Number Sense" and in the Glossary, that mathematical educators in California know what prime numbers are and tell us carefully. (cf. Pennsylvania, below.)

Scientific calculators are mandated for the first time in grade 6, but only for good reason, and after the essential properties of the real number system have been assimilated through hand calculations. (Japan introduces these electronic aids in grade 5, while most American states *demand* their use from the beginning.) By the end of grade 5, California students are to be able to do long division with multiple-digit divisors and to represent negative integers, decimals, fractions, and mixed numbers on a number line. By the end of grade 7 they graph functions, use the Pythagorean theorem, evaluate algebraic expressions, organize statistical data, and in general are prepared for high school algebra and geometry.

STATE REPORT CARD

California

I. CLARITY	4.0
II. CONTENT	4.0
III. REASON	4.0
IV. NEGATIVE QUALITIES	4.0
TOTAL SCORE (out of 16)	16
GRADE	A

The years 8-12 are described by subject: Algebra I and II, Geometry, Probability and Statistics, Trigonometry, Linear Algebra, Mathematical Analysis, and as the "Advanced Placement" subjects of statistics and calculus. The *Standards* is not prescriptive in its pedagogy. *How* the teacher, or the textbook, goes about the job is left to the discretion of the teacher or school district. A table is provided suggesting placement of this material by year, so that integrated curricula are possible by the same standards as the subjects would demand when taken in the form of courses. It is clear that a course-by-course program would place Algebra I in the 8th grade, Geometry in the 9th, and Algebra II in the 10th, completing the state-mandated curriculum for graduation.

Algebra I names 25 items, some necessarily mechanical and some with welcome attention to logical structure, including knowing the quadratic formula *and its proof.* "Practical" rate problems, work problems, and percent mixture problems are to be studied as well as the (impractical?) Galilean formulas for the motion of a particle under the force of gravity.

In Geometry, students must be able to prove the Pythagorean theorem and much else, including proofs by contradiction, and classical Euclidean theorems on circles, chords, and inscribed angles. "Geometry" also includes the basic trigonometry of solving triangles, and the properties of rigid motions in the plane and space.

Mathematical Induction is introduced in Algebra II, which material is apparently intended for the 10th grade or earlier. Experience will have to show if this and certain other ambitious demands are really possible, or should rather be left for the following years and a volunteer audience.

Other topics are either preparation for college work or are (these days) college-level work themselves (de Moivre's Theorem, the Binomial Theorem, Conic Sections with foci and eccentricities, etc.). We are not told how much of this program must be taken by *all* California students, or whether the courses named Trigonometry and Statistics are intended as options.

There is no mention of "shop math," "finite math," or "business math" (etc.), directed at the non-college bound student, or remedial courses for older students who have failed earlier. Perhaps the Mathematics Framework (see below), required by California law to be revised for 1999, will address these issues. The authors of this report did not downgrade California (or any other States) for this sort of omission, when the core curriculum is adequate and well stated. We do, however, expect that California will in due course find it advisable to add something appropriate to its "elective" curriculum at the high school level, as does, for example, Tennessee, which offers a branching of alternate tracks.

If teachers and textbooks can be found to carry it through properly, this *Standards* outlines a program that is intellectually coherent and as practical for the non-scientific citizen as for the future engineer. Whatever of "real-world" application school mathematics can have, is found here, set upon a solid basis of necessary understanding and skill. Initial reaction to the adoption of this document included a widespread apprehension that this "return to basics" represented an anti-intellectual stance: rote memorization of pointless routines instead of true understanding of the concepts of mathematics. The opposite is true. One can no more use mathematical "concepts" without a grounding in fact and experience, and indeed memorization and drill, than one can play a Beethoven sonata without exercise in scales and arpeggios.

There is always a danger that intellectually challenging material, be it in music, literature, or mathematics, will in the hands of ignorant teachers, or bowdlerized textbooks, become reduced to pointless drills. The history of American school mathematics in the 20th century has largely been a chronicle of conceding defeat in advance, teaching too little on the grounds that trying for more will fail. California is to be commended for taking up the challenge head-on, and announcing its intention in the clearest terms in its *Content Standards*.

It is the more curious, then, that the adoption of this *Standards* has been attended by an extraordinarily bitter public debate centering on their characterization as a reactionary document discouraging, rather than demanding, the "real understanding" of mathematics. An earlier version of this *Standards* had been composed by a special Commission on Standards which worked through most of 1997 on standards for English and mathematics, to be approved by the State Board of Education. The appointment of the Commission was itself extraordinary, and the consequence of public dissatisfaction with current teaching of core academic subjects.

One can no more use mathematical "concepts" without a grounding in fact and experience, and indeed memorization and drill, than one can play a Beethoven sonata without exercise in scales and arpeggios.

California by law publishes a Framework for mathematics instruction every seven years. Past Frameworks included standards of the sort under review here, along with pedagogical and administrative information, and the most recent was published in 1992 in the midst of enthusiasm, on the part of the school administrators, for the point of view represented nationally by the NCTM Standards of 1989, and often called "reform." (The "reform" trend had been visible in California even earlier.) Two foci of opposition soon appeared: "HOLD" in Palo Alto and "Mathematically Correct" in San Diego, both citizens groups (including mathematicians and engineers) publishing web pages designed to persuade readers that the "reform" represented by the 1992 Framework and its progeny, should be discarded in favor of something usually (though simplistically) called "traditional." In particular, they pointed to what they said were deteriorating scores of California children on national tests.

In a word, the anti-reform camp claimed "Johnny can't add," but instead spends his school time measuring playgrounds and talking it over with his classmates; and he uses a calculator when asked to multiply 17 by 10. Thus the new, *ad hoc* Commission on Standards was appointed, quite apart from the legally mandated Framework committee, to—in effect—adjudicate this controversy. The Commission, a citizens' commission not intended to be expert in mathematics (new standards for other core subjects were also part of its charge), took advice from experts of its own choosing and ended sharply divided. It voted by a large majority in favor of a document that it submitted to the State Board of Education on October 1.

The Board, which has final say, heard much public testimony, including opposition expressed by mathematicians, and rejected the draft. The Board used that draft, however, as a beginning for the very substantial revision it ultimately approved in December of 1997. That revision, which is the *Standards* reviewed here, was mainly prepared by a group of mathematicians at Stanford University, and its publication has generated more public controversy than anything seen earlier. Apart from segments of the public, two groups of professionals are now in contention: the mathematics *education* community, or a vocal part of it, against the *mathematicians'* community, or a vocal part of that.

(In the meantime, the Board has the task of reconciling the mathematics standards implicit in the new Framework, expected to be published in 1998, with the revised *Standards* under review here.)

Newspaper reports of the controversy make it apparent that those opposed to the Board's revisions, and who wish the Board to return to the document submitted to them by the Commission in October, include the California Superintendent of Public Instruction and at least one high-ranking official of the National Science Foundation. This party portrays this revised *Standards* as a return to the failures of past years, a document devoted to the mindless, pointless manipulation of outdated algorithms. The authors of this report believe such a characterization is mistaken, and that the mathematicians who participated in the final revision had no such intention, and their product no such result—except as poor teaching might make it so. It is to the better mathematical education of teachers that California (and the rest of us) must look for improvement of result.

Colorado

The best feature of the Content standards is the listing of topics *additional* to those demanded of all students. Under False Doctrine is the prescription of calculators and computers for whole number arithmetic at the K-4 level. The entire text takes up only 16 pages, but then is followed by an Index occupying three pages, surely inflation of a sort. The text is often vague, indefinite, or careless—thus at page 12 the reader finds this phrase: "solving real-world problems with informal use of combinations and permutations"; and in the Glossary, "Algebraic Methods" are defined as "the use of symbols to represent numbers and signs to represent their relationships" (p. 22). As to the former, why "informal"? And by the Glossary definition, algebraic methods could be exemplified by "2+3=5," since 2,3, and 5 are symbols and + and = are signs representing their relationships. But "algebra" means something more than symbolism. On page 14, this carelessly placed item for grades 5-8 occurs: "Solve problems using coordinate geometry." Taken seriously, this is something done in 12th grade pre-calculus courses, or in college, unless "problem" means something quite trivial. Such carelessness must be counted False Doctrine.

STATE REPORT CARD

Colorado

I. CLARITY	0.7
II. CONTENT	1.7
III. REASON	1.0
IV. NEGATIVE QUALITIES	2.0
TOTAL SCORE (out of 16)	5.4
GRADE	D

Connecticut

This August 7, 1997 document is marked "Second Draft," but will require expansion and definiteness of reference to be of any value. Reason is pushed to one corner of the "Geometry" page—at the 9-12 level, and among five other items—where readers are told, "Develop an understanding of an axiomatic system through geometric investigations, making conjectures, formulating arguments and constructing proofs." This recapitulation of the thousand-year long development of ancient Greek mathematics is unlikely in a high school, and should be replaced by a realistic set of standards for deductive geometry, if that is what is meant. As written it is too ambitious. On the other hand, the following, from level 5-8, is too childish: "Use real-life experiences, physical materials and technology to construct meanings for the whole numbers. . . ." By grade 5 the abstract notion of a whole number should long have been in mind, without blocks and calculators.

The *Framework* is too brief to contain much False Doctrine or Inflation. Still, standard 4, "ratios, proportions and percents," runs through all grades; under it, at grades 5-8, we find the following: "use dimensional analysis to identify and find equivalent rates"; and this, at grades 9-12: "Use dimensional analysis and equivalent rates to solve problems." Not much progress there. The entire page, of which the two quoted items form about 20 percent, is inflated by the strain of finding enough words to justify the existence of standard 4 altogether, for it surely does not deserve equal billing with (say) standard 9, "Algebra and Functions"—and even

STATE REPORT CARD

Connecticut

I. CLARITY	0.7
II. CONTENT	1.0
III. REASON	1.0
IV. NEGATIVE QUALITIES	2.0
TOTAL SCORE (out of 16)	4.7
GRADE	D

Standard 9 omits geometric series, the quadratic formula, and the binomial theorem.

Delaware

The 1995 text is brief, though augmented by pedagogical suggestions, and it covers K-10 only, as a guide to statewide testing at the grade 10 level. An "Appendix A" has been added to the state's Web site, giving an indication, all too brief, of what will be expected at grades 11 and 12, and including a welcome announcement that the "logical framework" of algebra, and deductive proofs in general, will form an important part of the program. However, this part is not yet as fully elaborated as the main text covering K-10. Here, where the standards themselves are vague, which is often, the suggested instructional activities generally illustrate a very minimal content. Projects, keeping journals, reporting to the class, and "real-life" applications are too often emphasized above intellectual content. The constructivist stance leads to instructions concerning student "exploration," sometimes producing mystifying open-ended demands, e.g., "Examine the relative effect of operations on *rational numbers" ("Number Sense," grades 6-8)*. Relative to what? Which operations? Why *rational* numbers?

Where items are less vague, they can describe a valuable curriculum—e.g., for "Geometry" at grades 6-8, "Use a compass and straight edge as tools for basic geometric constructions" (p. 51).

If this were accompanied by a lesson in the reasoning behind the validity of some such constructions, it would prepare for the logical analyses indicated for the grade 11-12 levels. But the very use of the word "tools" for the ruler-and-compass backbone of Euclid's geometry generates a misunderstanding of that ancient branch of mathematics. There is very little of Reason threaded through the performance indicators.

STATE REPORT CARD

Delaware

I. CLARITY	1.0
II. CONTENT	2.3
III. REASON	2.0
IV. NEGATIVE QUALITIES	2.5
TOTAL SCORE (out of 16)	7.8
GRADE	C

District of Columbia

There is here an occasional good idea, but such ideas are few. On page M-13 (grade 11 level), the reader discovers this: "Illustrate that a variety of problem situations can be modeled by the same type of function." As a summary of what can be learned from several years of progress in algebra, this request is exemplary, but the rest of the document simply does not present or even outline such a program. (Idaho—see below—posts the same demand, *verbatim* but for the word "recognize" in place of "illustrate.") On page M-9 occurs this example of Inflation: "Through the use of technology, students can experience a richer set of algebra experiences that allow them to investigate algebraic models at a conceptual level through representations in terms of graphs, tables, polynomials and matrices." Richer than what? Algebraic models of what? Without definition, such sentences do not guide instruction. And "matrices" at the 11th grade level are a disproportionate demand, where algebra does not include the quadratic equation or binomial theorem. While the D.C. document doesn't contain much under the heading Negative Qualities, it also contains very little that is positive.

STATE REPORT CARD

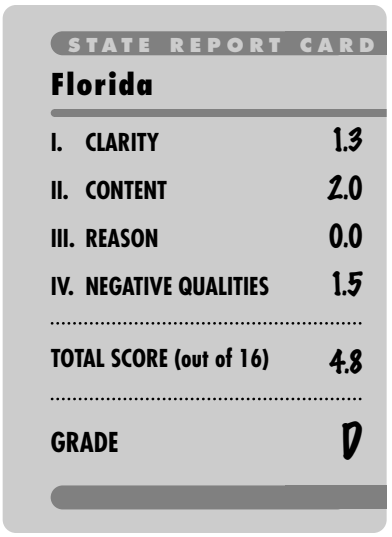
District of Columbia

I. CLARITY	0.7
II. CONTENT	0.0
III. REASON	1.0
IV. NEGATIVE QUALITIES	3.0
TOTAL SCORE (out of 16)	4.7
GRADE	D

Florida

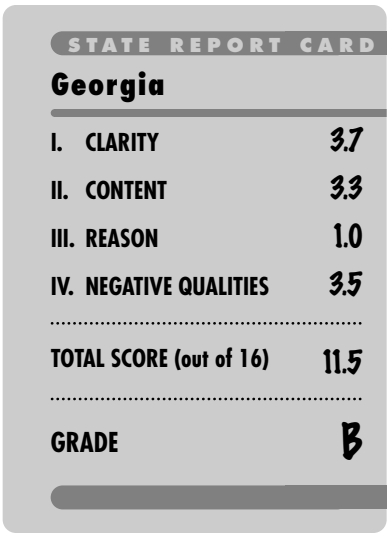
We consider the two documents (see Appendix) together: (1) T*he Sunshine State Standards* is filled with examples and teaching strategies as well as content advice. It is well printed and easy to use, but those parts covering grades 6-12 have been superseded by (2), *Florida Course Descriptions*, which describes the material by course titles rather than by "threads." The general tone of the *Course Descriptions* is consonant with that of the *Standards;* however, it features an overemphasis on the instrumental ("real-life") purposes and uses of mathematics, even in "honors" courses presumably directed at college preparatory students.

Thus, on page 3 of "Course 1206320-Geometry Honors," we see, at MA.B.3.4.1, "Solve real-world and mathematical problems involving estimates of measurements, including length, time, weight/mass, temperature, money, perimeter, area and volume, and estimate the effects of measurement errors on calculations." This is typical, and among other things rather careless in classifying money problems under "geometry." There is indeed very little geometry in this course, from either the Euclidean or the analytic standpoint. Some other failures: On page 112, we read, "Recognizes, extends, generalizes, and creates a wide variety of patterns and relationships using symbols and objects." This is not clear or definite, and is inflated. (The associated example is actually simple, and worthy of more explicit description.) On page 50, the Performance Description MA.A.1.4.3a, "determines whether calculated numbers are rational or irrational numbers"; it is exemplified by a contrived "real-world" situation concerning when an automobile's braking distance is given by a rational or irrational number. Real-world physics has not yet achieved the means of determining whether a measurement is or is not that of an irrational number. Thus, here is an example of straining after "real-life" ends by presenting a false scientific, if not mathematical, doctrine. Finally, on page 48 Example M.A.A.1.3.3a is a multiple-choice question with all its answer choices wrong: carelessness again.



Georgia

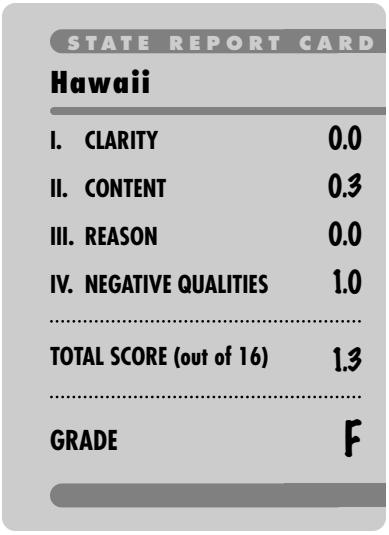
On page 1 we find that "[c]alculators and computers are essential tools for learning and doing mathematics at all grade levels." This is not so. The Georgia "Core Curriculum" is labeled "Draft," and is certainly one of the better ones; but it unfortunately avoids mention of the deductive structures of mathematics even in places where it would be natural as, for example, in "Geometry": "States and applies the triangle sum, exterior angles and polygon angle sum theorems." Why not ask for their proof, too? The proofs are not difficult, and are more enlightening than the results themselves. It is an opportunity missed. In another place concerned with geometry, the student is expected "to [use] tools such as compass and straightedge, paper folding, tracing paper, mira or computer to construct congruent segments, angles. . . ." This reduces ruler and compass to tools like the others, unrelated to the deductive structure of the subject. As tools, they are certainly inferior to computer printouts, but this is irrelevant to the instructional value that should be their purpose. This January, 1997 draft is not yet a completed document, especially in describing the organization intended for its advanced offerings, which are numerous and rich in content, and include Advanced Placement courses in statistics as well as calculus. It is mainly the standard K-12 curriculum implied by these standards that lacks sufficient attention to Reason, overemphasizes the uses of technology, and carries the message in all courses that mathematics is preeminently for direct practical use. Almost all the demands are clear and definite, but indefiniteness appears more often than it should. In "Discrete Mathematics," the student "solves problems that relate concepts to other concepts, and to real-world applications, using tools such as calculators and computers." Probably the authors had something definite in mind here, but the words do not convey it. Furthermore, too much is expected of technology here if calculators are expected to help "relate concepts to other concepts." This is something that should be done in the mind.



Yet much that is written is definite and good. For example, at the K-5 level, "Determines the missing number or symbol in addition or subtraction number sentences" describes a necessary ability exactly, and "uses skip-counting as readiness for multiplication" pays proper attention to the pacing of the teaching of arithmetic.

Hawaii

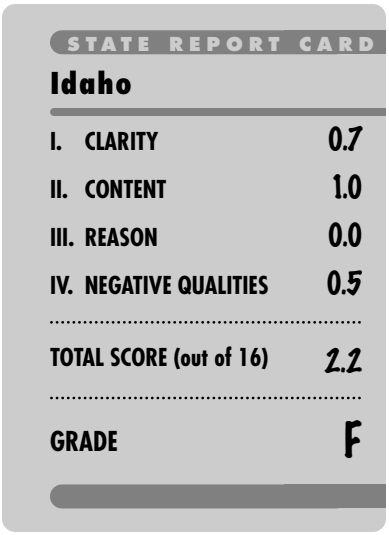
In the *Essential Content* document (2), there appears under "Content" in the "Statistics/ Probability" thread, the heading "Probability in real-world situations"; to its right, under the corresponding Performance rubric, are three entries, of which the third is "Solve real-life problems using statistics and probability" (p. 64). This is not an enlightening amplification of the heading. Both documents are of this vague nature. In the *Performance Standards* document (1), on page S-17, occurs the 9-12 instruction, "Geometric and spatial explorations include: . . . Non-Euclidean geometries, and hyperbolic and elliptical geometries." This is unbelievable where the documents do not otherwise mention axioms or proofs. And it cannot mean spherical geometry (see Note on Arkansas above), because, while spherical geometry is related to one form of non-Euclidean geometry, it is not at all hyperbolic. Very few statements in either document are definite enough to qualify as False Doctrine, or any doctrine at all.



Idaho

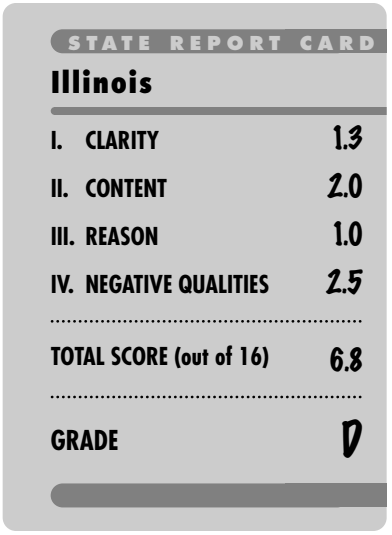
Grades 5-6 were plainly written by a different team than the others, and are much better than all the rest. We count grade 6 as part of "middle school" for scoring purposes here; hence, this part of Content receives a better grade than the rest, which is inadequate on all counts. The total organization of the *Content Guide and Framework* generates some careless repetitions—for instance, fractions and decimal calculations are prescribed in nearly identical terms at all grades from K through 4. Surely not "decimals" in kindergarten? And one "performance objective" at 1st grade level is that "[a]ll students will reflect on and clarify their thinking about mathematical ideas and situations." This is not testable.

Of the 9-12 level the reader is told, "Every mathematics course in Grades 9-12 will address objectives from each of the fourteen standards included in this framework. In order to accomplish this, tedious computations and graphical representations by pencil and paper and pencil drill must be de-emphasized to the point that the use of technology (calculators, computers, etc.) will be used to perform these tasks at all levels of mathematics." Yet Idaho is one of the few states to include "geometries" [plural] in its high school curriculum, asking all students to "develop an understanding of an axiomatic system through investigating and comparing various geometries." The unreality of this prescription is discussed above, where Arkansas and Hawaii make a similar demand. "Geometries" cannot be understood in the absence of the understanding of at least one example, preferably the Euclidean, something Idaho does not sufficiently offer. To speak in such grand terms, of "geometries," is Inflation. Under "Algebra" this standard appears: "demonstrate technical facility with algebraic transformations, including techniques based on the theory of equations," and under "functions," "recognize that a variety of problem situations can be modeled by the same type of function." The first of these statements is incoherent. (It also occurs *verbatim* in one of Arizona's "honors" performance objectives.) The second, which catches one's attention with the awkward and obscure phrase, "the same type of function," also occurs *verbatim* on page M-13 of the District of Columbia's *Framework.*



Illinois

Under Goal 7, dealing with measurement, at 7.C.4a, is an example of something clear and good: "Make indirect measurements, including heights and distances, using proportions (e.g., finding the height of a tower by its shadow)" is a plain and direct entry relevant to the Goal. Indefinite or obscure entries are much more numerous, however. For example, under "Number Sense," for late high school, the reader finds, "Determine the level of accuracy needed for computations involving measurement and irrational numbers" (p. 19). Again, 6.A.4, for early high school, states, "Identify and apply the associative, commutative, distributive and identity properties of real numbers, including special numbers such as pi and square roots"; this is quite incoherent, apart from the indefiniteness of "apply." "Calculators and computers" are apparently mandated in grades K-4 for teaching "number sense," among other things (Standard C, see p. 18), though the itemization is obscure on this point. This is either lack of clarity or False Doctrine. Certainly the standard algorithms for the decimal system are not prescribed. Here is excess verbiage: "Use geometric methods to analyze, categorize and draw conclusions about points, lines, planes and space." This can only mean "Learn Euclidean geometry," and indeed the document does call for the traditional two-column Euclidean proof format at 9.4.4c. But 9.4.4c is hard to take seriously in that it is confined to a single item among 18 such, the rest regarding geometry as mainly a practical and empirical study; and there is not enough time left to do all that is implied by "two-column proofs." There is, in general, a paucity of demand for deductive reasoning in other parts of the curriculum, too, though a wise teacher might read some such demand into the rather indefinite instructions that are given. The *Learning Standards* are avowedly intended for all students, but appear to be a compromise: over-ambitious for some and neglectful of others, and vague enough to permit many sorts of teaching and many depths of curriculum.

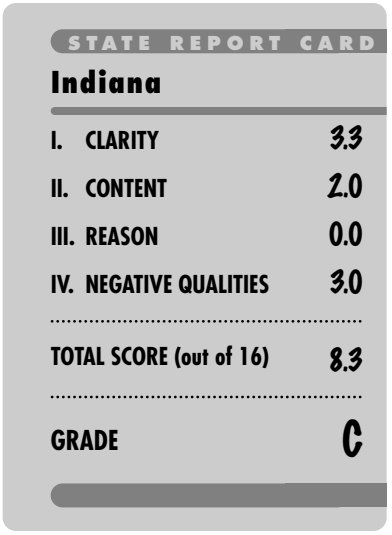


Indiana

The writing in this *Mathematics Proficiency Guide* is very good, direct and plain, and only sometimes indefinite, as when phrases like "in a variety of ways" are used. The content, however, suffers from an unusually heavy commitment to "real-life" applications as the rationale for all lessons. On page 207, "Constructive learning should be incorporated into almost every mathematics lesson" apparently requires the state to omit all too many mathematics lessons, as our ratings for Content indicate. The logical structures of algebra and geometry are among the omissions.

Even the announced commitment to "real-world" examples is sometimes belied by a predisposition to theory, as when a table of heights attained by an upwards thrown baseball turns out to be a theoretical set of heights attained by the Galilean formula for falling objects, and not a set of measurements as advertised.

There is also mathematical error, as in the "topology lesson" on page 154: "Give students a piece of paper with a circle the size of a dime. Cut out the circle. Can a quarter be made to pass through this hole without tearing the paper?" This is not a lesson in topology. Topology is, of course, not a K-12 subject, but if the word is mentioned it ought to be mentioned correctly. On the same page, "The formation of a sea shell depicts many golden ratios which are intertwined" is very badly stated. There is only one golden ratio, not that this fact is the only error in the quoted statement. However, the fact that the author here cannot or will not distinguish between a mathematical abstraction (the golden ratio) and its applications or exemplifications is characteristic of the view of mathematics inculcated by the entire *Proficiency Guide*. These are but a few examples of the plethora of "activities" and "real-world" applications that take up most of the space in this long document, in which actual demands for mathematical content take but a small space.



Kansas

This *Standards* is very well organized: the "Outcomes" are listed and classified by subject thread and grade level early in the document, then spread out, one "outcome" to a page, followed on that page by a number of examples or exercises illustrating their import, for they are rather general in language and tone. But the illustrations are often trivial and illustrate very little, or they are carelessly written, sometimes even suggesting lessons destructive of the announced purpose. On page 74 are given two matrices named A and B, with instructions to multiply to get AB and then BA, and draw some lesson therefrom. (This is a calculator exercise.). But the desired "lesson," which appears to concern the commutative law for real numbers, according to the top of the page, cannot be had because BA does not exist, i.e., the matrices cannot be multiplied in that order. People who know matrix multiplication only from calculator exercises, even written correctly, cannot learn the main lessons to be learned from matrix multiplication in school.

A more striking example of the inappropriate emphasis on technology occurs at the 8th grade level: "Use a computer program that generates Pythagorean triples. (Note: Many text-books have BASIC language computer programs already written to do this. The student could enter the program and run it.) What are the first 10 Pythagorean triples? (i.e., the 10 Pythagorean triples with the smallest unique whole numbers)"

The reader is immediately impelled to wonder whether the triple (20,21,29) "precedes" or "follows" (9,40,41), but this is incidental. The real fault here is that using a previously written BASIC program teaches a student no more about Pythagorean triples than would the teacher's writing some of them on the blackboard. A "lesson" of this sort is one consequence of a standards document's insistence on "appropriate technology" and manipulatives at all levels. Algebra tiles, which Kansas recommends for the 10th grade, are another. The 10th grade is a time when one hopes such "training wheels" have long been left behind. Here, instead, Reason is left behind. False doctrine.

STATE REPORT CARD

Kansas

I. CLARITY	1.7
II. CONTENT	1.3
III. REASON	0.0
IV. NEGATIVE QUALITIES	1.5
TOTAL SCORE (out of 16)	4.5
GRADE	D

Kentucky

Kentucky's companion document, *Transformations,* is not listed in the Appendix because it is a guide to instruction rather than a standards document of the sort we find in the *Core Content* commented on here. The latter is organized to show, for each thread (such as "algebraic ideas"), the items labeled "concepts," "skills," and "relationships," at each of the levels—elementary (K-5), middle (6-8), and high school (9-11)—or more accurately, what is expected by the time of assessment, which will be during the 5th, 8th, and 11th grades. The list is brief and generally inadequate. As a guide to assessment it would have to be augmented by many specifics, and perhaps experienced Kentucky teachers already know them; but the purpose of a *Core Content* guide is not simply to be a general reminder of what one already is doing. Nor is such a guide very helpful to new teachers.

A typical "skill" listing for "algebraic ideas" at the grade 5 level is, "Students should be able to find rules for patterns, extend patterns, and create patterns" (p. 6). At grade 11, they should be able to "solve and graph a variety of equations and inequalities." It is not possible to deduce from statements like this whether the content at each grade level is rich or poor, and we have rated Content accordingly, i.e., only by as much as we have been instructed by the text. Some instructions are more definite, and this pattern should be followed elsewhere—e.g., "Students should understand arithmetic and geometric means." This is good as far as it goes, but at such places, where the *Core Content* is definite, it is meager.

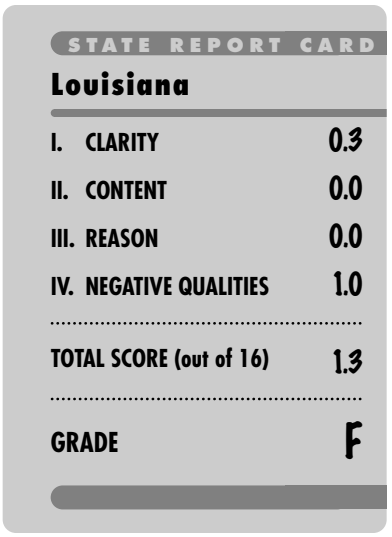
While a vague instruction might sometimes imply more content than our scores give credit for, one also sometimes finds that the vagueness is actually impenetrable, as when students are to understand "how ratio and proportion can be used to connect mathematical ideas." Ratio and proportion are relationships that in school mathematics connect numbers, or lengths, etc., not mathematical ideas. This must be called Inflation. Again, 11th grade students are to understand "order and equivalence relations," and "how numbers in the real number system relate to each other." Were the phrasing more comprehensible and definite, these might describe some rather advanced college work. The abstract idea of "equivalence relation," for example, and any application of that idea, are found difficult by many advanced college undergraduates.

STATE REPORT CARD

Kentucky

I. CLARITY	1.0
II. CONTENT	1.7
III. REASON	0.0
IV. NEGATIVE QUALITIES	2.5
TOTAL SCORE (out of 16)	5.2
GRADE	D

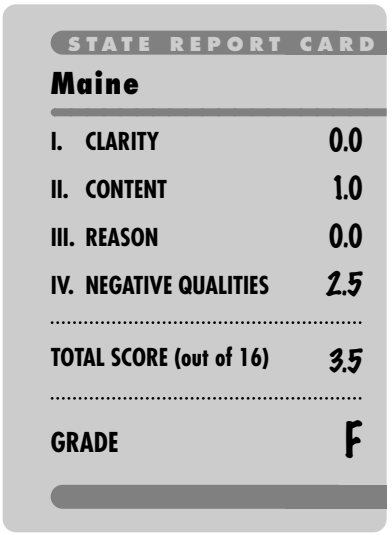
Louisiana



It is possible to find a meaningful and useful item here and there. At the 5-8 level, "demonstrating a conceptual understanding and applications [sic] of proportional reasoning (e.g., determining equivalent fractions, finding a missing term of a given proportion)" is perfectly plain and definite, and suited to the level. But consider Benchmark K-4, "Data Analysis":

"Demonstrating the connection of data analysis, probability and discrete mathematics to other strands and real-life situations." It is not possible to believe this is asked of the same K-4 students, less than 10 years old, who in another part of the same standards document are "identifying and drawing lines and angles and describing their relationships to each other and to the real world," something much more reasonable. At every level of this document, there is such a mixture of the trivial and the impossibly general or sophisticated—or opaque. Even the Glossary cannot be of real use to anybody; it defines "Coordinate Geometry" as "geometry based on the coordinate system," and "Magnitude" as "size or largeness." There is much else that is not helpful, including this: "Due to the rapid growth in technology, the amount of information available is accelerating so rapidly that teachers are no longer able to impart a complete knowledge of a subject area" (p. 1-2). Even without technology, the conclusion that more is known—about *anything*—than teachers can impart was true in the time of Plato. Such platitudes do not contribute to the purpose of a standards document, and are Inflation.

Maine



Under the Content Standard, "Students understand and demonstrate that ideas are more powerful if they can be justified," is the following Performance Indicator: "[Students should understand] that proving a hypothesis false (i.e., that just one exception will do) is much easier than proving a hypothesis true (i.e., true for all possible cases)." The lesson that a single counter-example falsifies a conjecture ("conjecture" is better than "hypothesis" here) is an important one, worth teaching. However, to cast proof as a means of making an idea more "powerful," rather than merely true, is not to make a mathematical statement at all, and perhaps to make a false statement, as the history of demagoguery shows. The teaching of Reason is ill served in general in these standards, as Maine's rating for Reason indicates.

This *Curriculum Framework* is for mathematics and science both, and gives the mathematical reader a chance to observe another field (science) in passing. On page 4, "Understand that matter can be neither created nor destroyed" is a doctrine of great 18th century importance, and was learned by children as late as 1900 in just those terms, but it is now known to be mistaken, and its exceptions have had important 20th century implications. With mathematics, on the other hand, the Maine document has it the other way round: the (apparent) *exceptions* to old doctrine are being emphasized beyond their desserts. Deductive thinking, given us by ancient Greece, is here replaced by a "content standard" urging that "students use different methods of thought to justify ideas," and one performance indicator under this standard advises that students "use intuitive thinking and brainstorming." These are things we do perforce; mathematics should teach us to govern these impulses, channel them, and recognize their pitfalls. Also under the general heading of Reason is "make inductive and deductive arguments to support conjectures." This is about as much detail as the document offers.

The illustrative, mainly "real-world," examples throughout the document do not often have much mathematical content. To advertise reasoning is not the same as to teach it in contexts where its value is visible. There is not much context here to make this possible.

Maryland

The *Framework* items of student "expectations" occupy only pages 18-23 of this 42 page pamphlet, the rest being introduction, educational philosophy, and other commentary. There are six extremely general Goals. For example, Goal 2 is "to develop an understanding of the structure of mathematical systems: concepts, properties, and processes." Each goal has four, five or six "subgoals." For example, Subgoal 2.3 is to "[u]nderstand concepts, properties, and processes of geometry," which is not much advancement in specificity. Finally, there are from two to 13 "expectancies" under each subgoal. These are also general, and not keyed to particular grade levels. Expectancy 2.3.7 is to "explore geometric ideas such as topology, analytic geometry, and transformations." This is as specific as the *Framework* gets, which means that it is of no practical value. This particular example was chosen to show that it can also be inflated and unrealistic: topology is no subject for school mathematics anyway, and this should be known to the person who wrote this "expectancy." Most of the *Framework* is of this level of generality and sometimes unreality, and while it also contains some advice about how local schools can use it in constructing curricula, it is not really a guide of the sort envisioned in the criteria for the ratings as described earlier in this report.

The most that can be said about change is that we now have some mathematical processes that were not earlier known; but the ones taught in high school have mostly been around for a long, long time, and the world still has need of people who understand them.

What value there is in the package of two documents in our possession is mainly in *The High School Core Learning Goals* (document (2) in the Appendix), which is more definite, but quite undemanding of content. It also has goals and expectations, e.g., under Goal 2, "Geometry, Measurement and Reasoning," Expectation 2.1 states that "the student will represent and analyze two- and three-dimensional figures using tools and technology when appropriate." One of the "indicators" under this Expectation (2.1.3) is that "[t]he student will use transformations to move figures, create designs, and demonstrate geometric properties." Clearly this indicator is compatible with whatever geometry course the local district might want to create. Despite the vagueness, there is False Doctrine in the overemphasis on calculators and computers, and the "real-world" applications, examples of which in the *Core Learning Goals* overwhelm whatever intellectual content the text might attempt to imply.

STATE REPORT CARD

Maryland

I. CLARITY	0.7
II. CONTENT	1.3
III. REASON	0.0
IV. NEGATIVE QUALITIES	1.0
TOTAL SCORE (out of 16)	3.0
GRADE	F

The two documents have the ambition to provide a philosophical background for curriculum and teaching, but do not end by saying anything that can be used, and by occasionally saying what is not so, as when the author of the *Core Learning Goals* makes this statement: "With the change in technologies, the mathematical processes change" (p. 1). How can this be, when on page 15 is pictured a house with a patio, inviting the student to find some optimal dimensions? This problem and its solution have not changed in 5000 years; though we now have a more convenient notation than did the ancients, the *mathematical processes* are the same. The most that can be said about change is that we now have some mathematical processes that were not earlier known; but the ones taught in high school have mostly been around for a long, long time, and the world still has need of people who understand them.

Massachusetts

The *Framework* is nicely laid out and printed, with excellent photographs as well as diagrams, but it is pedagogically and mathematically incoherent, and lacking in content. It says that manipulatives should not be confined to the early grades, and indeed it illustrates their use at all levels, but manipulatives are poor preparation for proofs "by mathematical induction" in the 9th and 10th grades (p. 78). Any college mathematics teacher will testify to the difficulty of teaching proofs by induction to college mathematics students, and the idea that this should occur in 10th grade is unrealistic, suggesting that "mathematical induction" has been confused with the rather vague notion of "inductive reasoning" in the mind of the author. Furthermore, the placing of mathematical induction under the heading of "Geometry and Measurement" is also incoherent.

At grade level 11-12, under "Geometry and Spatial Sense," we have this incoherent Learning Standard: "Use vectors, phase shift, maxima, minima, inflection points, and precise mathematical descriptions of symmetries to locate and describe objects in their orientation" (p. 81). The other standard in this section is "deduce properties of, and relationships between, figures from given assumptions," which is coherent but vague. Opposite these two, under "Example of Student Learning," is a picture of a parallel pair of Plexiglas rectangles held in place by some bolts, and the instruction to dip this into a soap solution to illustrate Steiner points for networks. This model has nothing to do with the two standards, and, except for its "popular science" appeal, has no educational value at this level of mathematical instruction.

At grades 5-8, students are to "engage in problem-solving, communicating, reasoning, and connecting to . . . develop and explain the concept of the Pythagorean Theorem." This kind of verbiage is inflated; the thing is a theorem, not a concept of a theorem, and "communicating" and "connecting" etc., are indicative of the sort of time-wasting richly repeated in other parts of the *Framework,* where "Examples of Student Learning" are given. All this in a document that doesn't ask for the essential skills of factoring of polynomials, or the quadratic formula, or the binomial theorem. Page 28 contains this: "Move away from the notion that basics must be mastered before proceeding to higher-level mathematics." For students of ordinary ability this is the road to "mathematics appreciation," not mathematics. Indeed, "mathematics appreciation" seems to be the main theme of the *Framework* which talks of Fibonacci numbers and Nautilus shells, soap films and Steiner points, fractals, notions whose mathematical content is generally beyond the school mathematics level, and which can only serve as amusements. The document as a whole is not serviceable for the purposes outlined in the introduction to the criteria, as given earlier in this report.

STATE REPORT CARD

Massachusetts

I. CLARITY	1.0
II. CONTENT	0.3
III. REASON	0.0
IV. NEGATIVE QUALITIES	0.0
TOTAL SCORE (out of 16)	1.3
GRADE	F

Michigan

The educational philosophy given in the introductory pages of the *Model Content Standards* is clear and definite: These standards are to be consistent with the "constructivist" view of the educational process, "firmly grounded in the work of John Dewey. . . . In other words, it is no longer sufficient to simply know mathematical facts; learners must be able to understand the concepts behind them and to be able to apply them to problems and situations in the real world." However, it is not possible to determine from the standards as written what are the "mathematical facts" desired, insufficient though they might be, let alone the way to "understand the concepts." The items are vague, and are more exhortations than standards, or are incoherent.

Under Content Standard 9, concerning number systems, at the High School level, students are to "develop an understanding of irrational, real, and complex numbers." There is something mathematically offbeat about the progression, "irrational, real, and complex," where the historical construction runs "rational, real, and complex"; and a mathematician's unease increases two pages later, where students are to "develop an understanding of the real and complex number systems and of the properties of special numbers including i, e, and conjugates." Conjugates? Again, this is an incoherent list, collected without thought of the pedagogical sequence it is intended to suggest, and perhaps without understanding of the non-parallel qualities of that particular listing.

A typical example of a standard is Content Standard 3: "Students develop spatial sense, identify characteristics and define shapes, identify properties and describe relationships among shapes." Below the standard are specifications, some for elementary, some for middle school, and some for high school. Here is one, for middle school: "Generalize the characteristics of shapes and apply their generalizations to classes of shapes." People who do not understand mathematics might be intimidated by the technical words "generalize" and "classes of shapes" into believing this sentence was written in English, or (alternatively) into believing that mathematics is not really supposed to be written in English, but either deduction would be incorrect.

The principal faults of the entire document are its brevity and its vagueness—a standards document cannot afford both—which render it an unsuitable guide to instruction or choice of curriculum.

STATE REPORT CARD

Michigan

I. CLARITY	0.3
II. CONTENT	1.0
III. REASON	0.0
IV. NEGATIVE QUALITIES	0.0
TOTAL SCORE (out of 16)	1.3
GRADE	F

Mississippi

This *Curriculum Structure* is well written and rich enough in content to serve its purpose, though it is sprinkled throughout with an excessive number of references to "real-world" applications, and is correspondingly short on instruction concerning the structures of mathematics (i.e., relative to our criterion, Reason). Page 51 contains this dangerous doctrine, though one observed by most other states without explicit mention: "Even though proof remains an important component in the geometric [sic] course, a shift from the traditional two-column deductive proof removes proof as the primary focus of the course to one in which the student provides informal arguments either orally or in writing." More happy, however, is the fact that Mississippi even mentions that proof (deductive proof) is important at all.

The exaggerated devotion to "real-life" applications sometimes leads to inflated claims, or verbiage, e.g., "The student will understand the role of application of matrices in connection with conic sections in describing real-life phenomena" (p. 65). Apart from the fact that no high school student is in a position to relate the mathematical properties of the conic sections with real-life phenomena, and that the use of matrices in their analysis is quite an advanced branch of algebra, this particular instruction is found under the goal concerned with statistics and probability, with which it has no discernable connection. (The conic sections are indeed the paths of comets and planets, but analyses of this depth cannot be done without some knowledge of calculus, which is not expected here, and is not particularly aided by the use of matrices anyway.)

Another contrived "real-life" connection is the following: "Solve problems involving factors, multiples, prime and composite numbers; include concepts of common factors and multiples and prime factorization (expressed using exponents); include real life applications of these concepts" (p. 33). The entire instruction is admirable except for the last clause, which is impossible.

STATE REPORT CARD

Mississippi

I. CLARITY	3.7
II. CONTENT	3.7
III. REASON	2.0
IV. NEGATIVE QUALITIES	2.5
TOTAL SCORE (out of 16)	11.9
GRADE	B

Missouri

By making some inferences that might or might not be consistent with the intent of the rather unclear wording of this entire document, one can deduce that Missouri intends a fair amount of mathematical content, which gives this *Framework* a better score for Content than under the other criteria. The document is well organized, not only by strand (e.g., "Data analysis, Probability and Statistics") and grade level (K-4, 5-8, and 9-12), but by "What all students should know," "What all students should be able to do," and "Sample learning activities." But these carefully constructed categories are not often filled with valuable information.

Under "Mathematical Systems and Number Theory," Grades 9-12, all students should be able to "select and apply appropriate technology as a problem-solving tool to achieve understanding of the logic of algebraic and geometric procedures" (p. 64). However, the *Framework* does not mention axioms and theorems with proofs as another way (i.e., in addition to "technology") of achieving this understanding. There is at this point a reference to NCTM Standard 14 which, upon examination, shows no mention of technology in this connection. The implied technology apparently includes "algebra tiles," which are mentioned on page 64, too, though they are better described as manipulatives. This is False Doctrine by our criteria, and an avoidance of the sort of mathematical instruction that should be implied by the title "Mathematical systems and number theory" that heads the page.

Other instructions on the same page are less mischievous because less definite: "Compare and contrast the real number system and its various subsystems," and "extend understanding and application of number theory concepts." Such language characterizes the *Framework;* here is another example taken almost at random: "Experiences should be such that students use discovery-oriented, inquiry-based and problem-centered approaches to investigate and understand mathematics" (p. 9). One can hardly select textbooks on the basis of this sort of instruction without already knowing from other sources what is to be taught.

STATE REPORT CARD

Missouri

I. CLARITY	0.7
II. CONTENT	2.0
III. REASON	0.0
IV. NEGATIVE QUALITIES	0.0
TOTAL SCORE (out of 16)	2.7
GRADE	F

Montana

The major part of the cited document concerns institutional matters: accreditation, teacher education, and general goals (and visions) for education in science and mathematics. The "Mathematics Curriculum Standards" occupy page 57 and part of page 58, and are mostly too general to be of use—e.g., "include the study of trigonometry." Brief and general as they are, the listed items exhibit a disinclination to ask that anything be overtly *taught,* suggesting instead "explorations" and "experiences," as in the instruction for grades 5-8 to "include explorations of algebraic concepts and processes."

STATE REPORT CARD

Montana

I. CLARITY	0.0
II. CONTENT	1.0
III. REASON	0.0
IV. NEGATIVE QUALITIES	0.5
TOTAL SCORE (out of 16)	1.5
GRADE	F

Nebraska

Page viii states this directive: "The mathematics/science learner will demonstrate mathematical and scientific literacy in a global environment." At page D-5 (high school geometry) occurs the "measurable performance" demand: "Use the deductive nature of geometry to solve problems." This demand is, as is the style of this *Framework,* followed by an entry under the follow-up rubric, "A closer look," urging real-life experiences to "enhance the understanding of modeling as a problem-solving tool." Finally, there are the "sample investigations" to close out this instruction: The first is a treasure hunt, and the second concerns the decorating and furnishing of a room. All told (and "global environment" or not), none of this helps "demonstrate mathematical and scientific literacy." The idea that deduction, allegedly asked for in this "measurable process," is a *mental* process is lost in all these "experiences." Another example occurs on page D-11: "Technology . . . must be used to build the basic notion of a function." Here the false doctrine is more urgent. Technology is no longer an aid, but a necessity, or so it is said. But the basic notion of a function was understood well before the technology required here was invented. Indeed, reasoning about functions helped develop technology, not the other way round.

STATE REPORT CARD

Nebraska

I. CLARITY	0.0
II. CONTENT	1.0
III. REASON	1.0
IV. NEGATIVE QUALITIES	1.0
TOTAL SCORE (out of 16)	3.0
GRADE	F

New Hampshire

This Framework is easy to handle and read, for it spends a bare four pages explaining what it is about, and then goes about it. The "Societal goals" are excellent, e.g., "All students will develop strong mathematical problem solving and reasoning abilities" (p. 3). Its page on "How students learn mathematics" quite properly emphasizes students' engagement with the material, and does not consider "real-life" applications the only path to knowledge. However, it repeatedly prescribes manipulatives for students at levels that should be beyond that sort of thing (False Doctrine), as at "Spatial Sense," on page 19, where the "proficiency standard" for End of Grade 10 says, "Use manipulatives, and/or coordinate geometry to explain properties of transformations. . . ." At this level, geometry should be presented conceptually, and indeed logically, rather than as a continuation of the wooden block constructions of kindergarten.

By the end of grade 6, this 30 page document uses three lines under "Number Operations" to ask students to "demonstrate an understanding that when dividing two whole numbers that are greater than one, the quotient will be smaller than the dividend." Such a detail is jarring in a list which, a few lines earlier, rather more grandly asks *3rd graders* to "explain the relationship among the four basic operations."

Another curiosity, unrelated to our numerical scores under the criteria listed above, occurs at algebra level 7-12, where students are to, "[m]odel and solve problems that involve varying quantities with variables, expressions, equations, inequalities, absolute values, vectors, and matrices" (p. 25). This sentence is a bit awkward, and would attract the attention of any mathematician by its inclusion of the misfit phrase "absolute values," which is simply not parallel with the other items named in a list already too-long. The word "with" is also strange here, but apparently means "using," as in New Jersey's Standard 4.13 for 12th grade algebra, which reads, "Model and solve problems that involve varying quantities using variables, expressions, equations, inequalities, absolute values, vectors and matrices," which (by what cannot be a mere coincidence) repeats the same list in the same order. Believing there must be a common origin to this rather strange listing, the authors went to the NCTM 1989 Standards, which is the acknowledged model for so many of the states' standards, and found on page 150 a similar, if simpler sentiment: "Represent situations that involve varying quantities with expressions, equations, inequalities, and matrices." The NCTM list, however, omits the misfit phrase "absolute values" and the word "vectors" that both New Hampshire and New Jersey chose to include at identical points in their identical listings.

STATE REPORT CARD

New Hampshire

I. CLARITY	1.7
II. CONTENT	2.0
III. REASON	1.0
IV. NEGATIVE QUALITIES	2.5
TOTAL SCORE (out of 16)	7.2
GRADE	C

New Jersey

The *Framework* (document [2], see Appendix) is a very ambitious and very long commentary on the *Content Standards* (document [1], which appeared a little earlier). A teacher who follows the outer implications of the *Framework's* many suggestions could very well prepare a student to bypass the first year at MIT, while others, using other textbooks equally consistent with these standards, would be able to avoid much that should be mandatory in any school system. More than most states, New Jersey seeks to embed mathematics in a cultural frame of reference, which is a worthy effort in the hands of well-educated teachers; but the classical content of mathematics, and its backbone of deductive reasoning, without which no amount of cultural framework can really be understood, are often slighted in these standards.

A teacher who follows the outer implications of the Framework's many suggestions could very well prepare a student to bypass the first year at MIT, while others, using other textbooks equally consistent with these standards, would be able to avoid much that should be mandatory in any school system.

No proof of the Pythagorean theorem is demanded, yet the student is expected to "explore applications of other geometries in real-world contexts" (Standard 7, grades 9-12). While this is not explained in the standards, the *Framework* gives as an example of "other geometries" the thoroughly Euclidean geometry of figures drawn on a sphere. (There really are other geometries than the Euclidean, such as projective geometry; and the discovery of hyperbolic geometry in the 19th century was a milestone in the history of mathematics.) Without a good background in the Euclidean deductive system, or of more analytic geometry than any high school is likely to offer, such "non-Euclidean" studies are more "math appreciation" than mathematics.

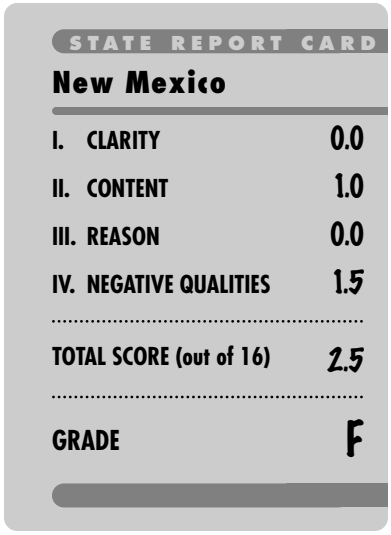
The emphasis on "real-world" applications and activities is extreme, and one must often guess at the intellectual content implied. In other places, there are admirable instructions—e.g., "Describe and apply procedures for finding the sum of a finite arithmetic series and for finite and infinite geometric series," which would be even more admirable if the word "prove" had appeared somewhere in that sentence. There is some attention to Reason in the New Jersey standards, but not enough, which is one concomitant of the falling off of Content in the high school years.

STATE REPORT CARD

New Jersey

I. CLARITY	2.0
II. CONTENT	2.7
III. REASON	2.0
IV. NEGATIVE QUALITIES	2.5
TOTAL SCORE (out of 16)	9.2
GRADE	C

New Mexico



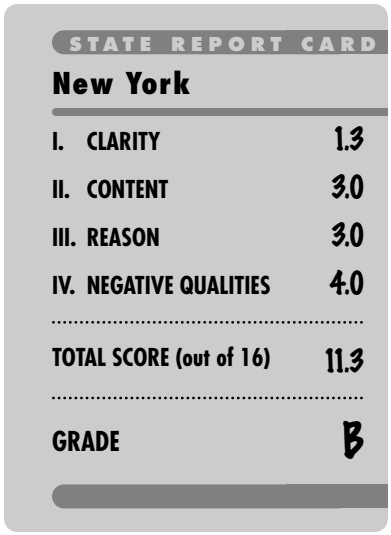
One good thing about these standards is the forthright statement at the outset that "proficiency in English is of the highest importance." This is immediately qualified, however, by supporting "the use of the student's primary, or home language, as appropriate . . . while the student acquires proficiency in English." There is not much to object to here, but there is also not much guidance, as to when and how much and how.

Similarly, Standard 1 ("Problem solving") states that "students will use manipulatives, calculators, computers, and other tools, as appropriate, in order to strengthen mathematical thinking, understanding, and power to build on foundational concepts." This instruction is given *verbatim* for each of the levels K-4, 5-8, and 9-12. But the evasive "as appropriate" renders this recommendation nugatory, just as in the earlier recommendation concerning bilingual education. Teachers are looking to the state's standards exactly to find out what is appropriate and when; it is not for the state to leave the essence of its recommendations undefined. Instructions so heavily qualified as to be interpretable at will must count as Inflation. And if the interpretation is the use of manipulatives in high school, or calculators at K-4 when arithmetic begins, it is False Doctrine.

The language of the *Standards* is sometimes garbled by excessive devotion to generalities. Standard 2 asks that students "use mathematics in communication" where the most likely interpretation, as evidenced in the associated benchmarks, is that "use communication in mathematics" is closer to the intention. The NCTM 1989 Standards has a thread called "Mathematics as Communication," which is probably closer yet. A similar inversion takes place in Standard 3, which states that "students will understand and use Mathematics in Reasoning," apparently a misprint for ". . . use reasoning in mathematics," since at least two of the associated benchmarks concern the use of reason in obtaining mathematical results.

Even so, the specifics are vague, here and throughout the document. Under "Number systems and number theory," for example, appears the line, "develop and analyze algorithms." Specifications this general cannot serve as a guide to teaching or curriculum. More general yet, indeed breathtaking, is this one, under Standard 4, for grade Level 5-8: "Students will apply mathematical thinking and modeling to solve problems in other curriculum areas such as employability, health education, social studies, visual and performing arts, physical education, language arts, and science; and describe the role of mathematics in our culture and society." Among other things, this particular skill is hardly testable.

New York



The best feature of this booklet, in which only 16 pages are devoted to mathematics, is the absence of inflated verbiage and unrealistic expectations, and the absence of downright bad advice. However, the brevity of the mathematics section of the "mathematics, science, and technology" standards permits extremely varying interpretations as to content. On page 26 it asks for proofs by mathematical induction, for example, a difficult topic at the high school level, and it asks for the analysis of infinite sequences and series. These are quite demanding and specific. Yet at the intermediate level the requirement that students "develop appropriate proficiency with facts and algorithms" (this comes under mathematical "operations" as a rubric) permits too much leeway. Sometimes only a teacher already experienced in what New York means by certain terminology will catch the drift of a too-condensed demand—e.g., at high school level, "Students . . . model and solve problems that involve absolute value, vectors, and matrices" (p. 27). Any mathematician would be puzzled at this juxtaposition of vectors and absolute values. And, in the same list of performance standards, "represent problem situations using discrete structures such as finite graphs, matrices, sequences, and recurrence relations" seems to imply a lot of content, but it does so somewhat vaguely, in view of the phrase "represent problem situations," which does not quite say what it probably means. Does it mean "solve problems," and if so, what sorts of problems? Here is a place where there is white space on the page, in which illustrative exercises would be valuable; but in general, the illustrative examples are few, and most of them are on the whole much less demanding than any reasonable interpretation of the text.

North Carolina

There is very little to complain of in this generally excellent document, from which other states could learn. It is carefully done. It has over 500 pages, but they are occupied with practical information of exactly the sort the new teacher in town would need to know. Only pages 13-61 are required to outline the content, but it is done briskly, by topic and grade level. Most of the rest of the book reiterates the content standards and couples them with "sample measures," which are test questions or classroom exercises. These can serve a teacher either as class material or, most important, as clarification of the import of the standards themselves.

Not everything clear and definite is necessarily good, however. On page 55, competency goal 5.2, "Use synthetic division to divide a polynomial by a linear binomial," betrays a rather old-fashioned origin for this Algebra II curriculum. Synthetic division is a superb example of an unnecessary algorithm, one which, unlike the "long division" algorithm for the 5th grade in the hands of a good teacher, does not offer any insight into the nature of the process. Such time-wasting is unusual in this curriculum, however.

There is very little to complain of in this generally excellent document, from which other states could learn.

There is nothing dated about 7.11: "Explore complex numbers as solutions to quadratic equations" (p. 55). Filled in with good teaching, the instruction is excellent (though "explore" ought not to be called a "competency goal"), but will this rather general "exploration" lead to learning? On page 472 this goal is repeated, with some "sample measures," among them, "Find a quadratic equation that has these roots: 5+2i and 5-2i"; and, "solve the quadratic equation . . ." [here an example requiring the quadratic formula is given]. However, here as in all too many other places, the inner structures of mathematics are ignored in favor of a telegraphic crispness. It would seem exemplary to say, "Here is the equation; find the roots," and then, "Here are the roots; find the equation." But there is more that can and should be said: What if the given roots are not complex conjugates of each other? Does the sum of the roots have the same connection to the coefficients in the complex case as the real? What *good* is a quadratic equation?

It is hard to introduce such considerations into a list of contents, and most states have gone overboard trying to do so, generally to the disadvantage of the contents themselves. Yet it can be done, and the places where North Carolina's scores are less than 4 are indications of such minor failures.

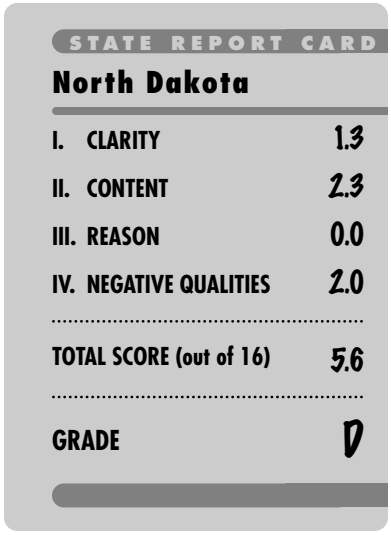
Reason, for example, gets a good workout in the geometry course, but somewhat less in the algebra and trigonometry sections, where it could have been injected with little extra effort. The excessive number of threads in the elementary grades leads to a diffusion of content. And in the "sample measures" sections that follow the content listings, a disproportionate amount of space is devoted to the 9-12 segment of the curriculum. Elementary teachers, who are generally not specialists in mathematics, need more guidance, year-for-year, than those in the more advanced years.

STATE REPORT CARD

North Carolina

I. CLARITY	4.0
II. CONTENT	3.7
III. REASON	3.0
IV. NEGATIVE QUALITIES	3.5
TOTAL SCORE (out of 16)	14.2
GRADE	A

North Dakota



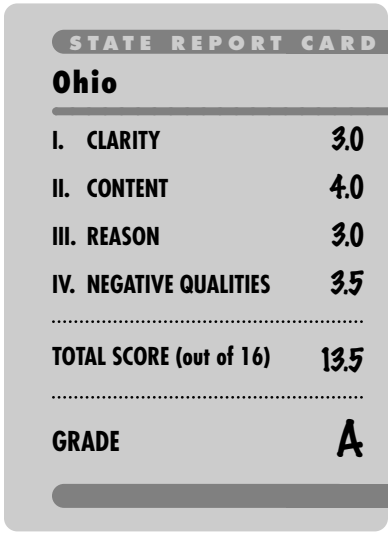
The organization is excellent, each page containing one standard (e.g., "Number Theory" or "Geometric Concepts") and one of the three grade levels K-4, 5-8, or 9-12. Then "Benchmarks/Performance standards" give some particulars, and are followed by examples of "specific" knowledge that support them. Finally, there are "performance activities," exercises or classroom projects, to illustrate the knowledge to be acquired, and how.

It is to be expected that the standards are quite general, but the benchmarks are often not much more particular, while the "examples of specific knowledge" are usually lists of vocabulary items with little indication of how much is to be known concerning each. For example, under "Data Analysis" at the 5-8 level, two of the benchmarks are "evaluate arguments that are based on statistical claims" and "display and use measures of central tendency and measures of variability." As written, these could as easily be placed at the 9-12 level, or in college, and indeed some of them do reappear at 9-12. The reader has little idea what progression in knowledge is contemplated as the student grows older, i.e., what is written is not sufficiently definite and is not testable.

Finally, the associated activities often are trivial, and not up to the intellectual level suggested by the benchmarks, both here and elsewhere. In the present case, one of them reads as follows: "Students work with partners to measure one another's height in centimeters. These heights are recorded on the board. Each student is to find the mean, mode, range, and median of the heights." As this could be done with any set of numbers, nothing was gained by the time consuming use of teams and yardsticks. (In the following activity, by the way, where predictions are called for at a somewhat higher intellectual level, the data set is hypothetical.)

Much of the document is good, but the intellectual level deteriorates in the higher grades. There are brave words— e.g., in 9-12 algebra all the elementary functions, all of them, are to be "investigated," but it is not clear how deeply or to what end. One cannot determine from these standards whether students learn the binomial theorem or the proof of the quadratic formula or Pythagorean theorem. The geometry course in particular shows little logical structure or coherence. Yet "Fermi problems" are mandated at page 35, "discovering fractals" at page 29, and "coding theory" at page 31. "Fractals" and such things as "conjecture" are defined in the Glossary, but Fermi problems and coding theory are not. (The Glossary contains some careless errors, too.) At the grades 5-8 level students are to "solve problems using coordinate geometry" (p. 21). This is not believable, unless by "problem" is meant something like plotting a point whose coordinates are given. Some of the performance activities that illustrate this section on geometry require the use of "multi-link cubes," marshmallows, toothpicks and a "geoboard."

Ohio



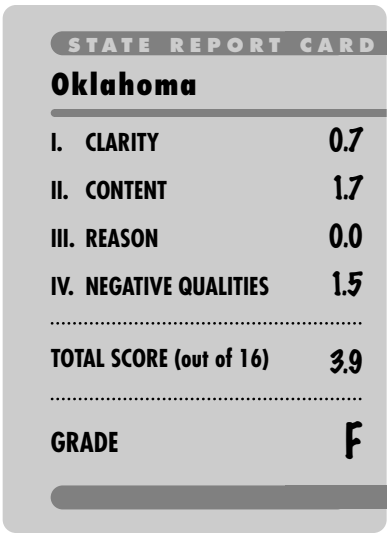
Although it features extensive (attributed) quotations from the NCTM 1989 Standards, this document's prescriptions are usually more definite and comprehensive. From the 10th grade onward, each mandate is followed by a section headed, "In addition, college-intending students will . . .," the section containing extra or more advanced material. Ohio is therefore among that minority of states that recognize in their standards that it is not wicked to offer some students opportunities not every student might desire or be able to use.

The organization is by strand and level, the levels being keyed to what are apparently state-wide "proficiency tests" given at the 9th and 12th grades, as well as some earlier standardized achievement tests at grades 4, 6, and 8. Most items are introduced with "The student will be able to . . .," and the rest is brief, e.g., "identify common shapes in the environment" (at the K level), or "identify parallel lines, perpendicular lines, and right angles in geometric figures and the environment" at the grade 4 level. The wide margins have space for clarifying comments here and there, and suggested exercises or illustrations.

The standards take less space than appears in this over-200 page book, as they are repeated, first classified by strands and then by grade level. In three pages, then, one can see what is expected by grade 5 in all subjects. Alas, one of the "subjects" in the early grades is Strand 2: "Problem-solving strategies," which contains largely vacuous demands, such as "validate and generalize solutions to problems" (p 133). Still, even in Strand 2 there may occur a valuable reminder of something not every state demands, e.g., (still at 5th grade level), "Read a problem carefully and restate it without reference to the original problem" (p. 33).

In Negative Qualities, Ohio is slightly downgraded in the category False Doctrine for its too frequent use of "explore" when naming something a student should be able to do. Anyone can explore, after all, and a teacher using these standards might well slight the direct transmission of knowledge, and demand for its exhibition, in favor of open-ended discussion, some of which is valuable in its place, of course, but the place of which should be better defined than it is in this document.

Oklahoma

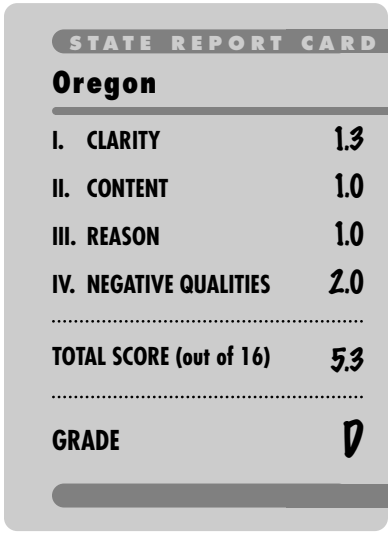


These *Priority Academic Student Skills—Mathematics* employ the categories (or threads) of the NCTM 1989 Standards, except at the kindergarten level, where it wisely recognizes their inapplicability, and simply lists quite reasonably what in mathematics should be done there. It would have done well to continue in this vein, for the NCTM threads, "Problem Solving," "Number Sense," "Patterns . . .," etc., often encourage the invention of content items that make little sense in many grade levels. In the early grades, however, the items are often clearly and definitely stated, as at the grade 1 level, "Use models to construct addition facts to 10." By the 9-12 levels, however, such a statement as "Recognize the connections between trigonometry, geometry and algebra" is surely inflated, and surely untestable, as is "Understand the connections between trigonometric functions and polar coordinates, complex numbers and series" (p. 49), something that (though also vaguely stated and untestable) could only be appropriate in a calculus course, so far as "series" is concerned. (This last list of things to be "connected" also appears *verbatim* in the South Carolina *Framework,* page 144.) Vast and inappropriate generalities of this sort must leave a teacher wondering about his own competence, when it is really the author of the lines who is prescribing impossibilities.

One curiosity: The resemblance between these core skills and the NCTM Standards is reinforced by the unattributed quotation, on page 48, of a line that appears on page 150 of the NCTM Standards: "Represent situations that involve variable quantities with expressions, equations, inequalities and matrices," a strikingly awkward formulation that appears in a slightly varied and even more awkwardly augmented version in the New Jersey and New Hampshire standards.

As to Content and Reason, there is in the present document no mention of the Pythagorean theorem, or of any particular axiom or theorem connected with geometry, which is still said to be studied "from several perspectives" and whose "foundations (e.g., postulates, theorems)" are to be understood "through investigation and comparison of various geometries" (p. 48). The surrounding text does not support such ambition, even if it were possible at the high school level. Nor is the logic of algebraic procedures delineated either by prescription or by implication. The general vision of mathematics offered is lacking in both substance and coherence, and is poorly expressed as well.

Oregon



The "Oregon Standards" named in the Appendix is a very brief portion of a very brief document of that name, and by itself does not serve the present purpose. However, the state sent, in answer to our letter of inquiry, the *Test Specifications for Statewide Examinations* at the 5th, 8th and 10th grade levels, along with sample tests at those levels and a *Teachers' Support Package.* Taken together, our hypothetical newcomer to Oregon has here the equivalent of the more usual form of curriculum standards for guidance; and we felt entitled to rate the results along with those of other states which provided the more usual standards or framework documents, recognizing that the materials at hand are intended for all students, and only up to the 10th grade level.

The best quality to be observed (especially in the *Teacher's Support Package*) is the encouragement given students to write their answers to questions in good English sentences. However, the content demanded is disappointing, and almost nothing is asked for that answers our criterion concerning Reason. The grade 10 examination booklet states (under "Measurement,") that "students will be given necessary formulas to complete problems" in the examination room (p. 6). The eligible content includes all the common figures, including circles. In other words, the basic competence demanded of Oregon high school graduates includes none of the common mensuration formulas, let alone anything of theoretical import. At the same level, "Geometry" contains an exercise in recognizing which of four drawings depicts a pair of perpendicular lines and which does not, surely an exercise in mere vocabulary, and that of a 5th grade level. The Pythagorean theorem is mentioned, but not in any significant context.

Altogether, an undemanding curriculum is implied by both the very brief "standards" and the more extensive but excessively simple *Test Specifications.*

Pennsylvania

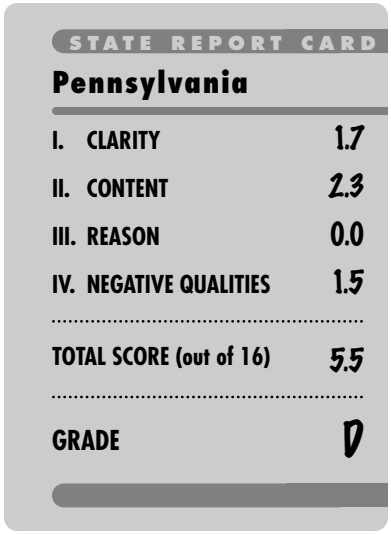
While the "Proposed Academic Standards" examined is a provisional document ("proposed" by the Department of Education for approval by the Governor's Advisory Commission on Academic Standards, that is, with further approvals before becoming official), the introductory commentary says that "the suggested standards contained in this report are the product . . . of more than a year of work by educators and others from all across Pennsylvania, the comments and suggestions of literally hundreds of citizens from all walks of life. . . ." The Commission did not write the document, but "the members of the Commission feel the standards being recommended here are rigorous, measurable, applicable, and understandable expectations of what students should know and be able to do." Also, "The Commission was diligent in its efforts to remove from the standards educational and professional jargon."

The result, as evidenced by a comparison between some items in this document and their correspondents in an earlier and controversial standards document of 1993, is indeed less jargon, but not, we believe, what the state hoped for. The Glossary defines "prime" so as to include "one" as a prime, and states that "1, 2, 3, 5, 7, 11, 13, 17, and 19 are prime numbers" (p. 27). "Gradient" is also incorrectly defined, and the definition of "limit" exhibits a familiar obscurity. The deficiencies in the standards themselves are congruent with these observations. The introduction states: "While mathematics is a very interesting and enjoyable field to study for its own sake, it is more appropriately used as a tool to help organize and understand information from other disciplines." We submit that this is a false dichotomy, and that the use of mathematics is appropriate everywhere.

The standards themselves are divided by grade levels: 3,5,8, and 11 marking the levels of what seem to be proposed statewide assessments. They are also divided by threads, e.g., "Number Systems . . .," "Problem-solving . . .," "Probability . . .," etc.

The attempt to avoid jargon is palpable and often successful, so that the items are brief. Thus, the reader finds, under "Number Systems" at grade 3, "Use drawings, diagrams, or models to show the concept of fraction as part of a whole"; at grade 5, "Explain the concepts of prime and composite numbers"; at grade 8, "Use models to represent operations on positive and negative numbers"; and at grade 11, "Convert between exponential and logarithmic forms." Important vocabulary items are italicized where they appear, which is itself a guide to the intention of the phrase or sentence.

Yet jargon of a sort remains, inherent in the way the ideas are expressed and not only in the words. "Convert between exponential and logarithmic forms," once its meaning is penetrated, says no more than that the definitions of "exponent" and "logarithm" should be understood. The awkwardness of the "convert between" phrasing here derives from a long tradition of confused jargon whose usefulness was finally obliterated by the modern conception of the exponential functions and their inverses. In 1940 (already anachronistically) the schools taught that "a logarithm is an exponent. . . ." This is not actually mistaken, but it is, or was, the beginning of a tangled sentence that most high school students of that era failed to understand. The current attempt, quoted above, may be shorn of most of that tangled wording, but it still betrays the lack of change of concept. By grade 11 a student should either learn nothing at all about logarithms, or should learn about inverse functions at the same time, to be able to place the subject in its reasonable setting, a setting not recognized by the quoted item.



In short, the content implied by the entire document is less than what students can and should learn at each grade level; even if the jargon used earlier has been improved, the deficiencies once made easily visible by such language too often remain. On the other hand, the text sometimes announces a startlingly ambitious curriculum expectation, as where it asks grade 11 students for proofs by "mathematical induction." The average grade 11 student will not learn this unless thoroughly educated in other forms of mathematical reasoning and syntax at earlier stages of the curriculum, and in the context of definite subject matter, such as algebra and geometry. But the entire thread containing mathematical reasoning (p. 7) exists in isolation from the others, which show little evidence of demand for reasoning within substantive contexts. Here the Pennsylvania document exhibits what is probably the most common failing of all the standards documents this report has studied.

Rhode Island

This attractively packaged *Framework* has a small poster-sized full text of the standards folded into its inside front cover, all of which is repeated in the *Framework* proper, along with commentary and pedagogical notes. The notes contain a philosophical view that partially explains the vagueness of the standards themselves, but do not explain what in the way of mathematical demands upon the students they are supposed to represent.

Sample entries: Under "Algebra" (grades 5-8), "Identify and justify an appropriate representation for a given situation." Under "Reasoning" (grades K-4), "Make logical conclusions about mathematics." Under "Problem-Solving" (grades 11-12), "Use sophisticated as well as basic problem-solving approaches to investigate, understand, and develop conjectures about mathematical concepts." None of these is amplified anywhere in the document.

As philosophical background, which we count False Doctrine: On page 15 are listed features of "the traditional classroom" opposite a corresponding list for "the learning community," which Rhode Island intends shall replace it. Opposite "Teacher knows the answer" is "More than one solution may be viable and teacher may not have it in advance." Opposite "Thinking is usually theoretical and 'academic'" is "Thinking involves problem solving, reasoning, and decision-making." We believe thinking involves all that is named on both sides of this apparent dichotomy and more, and deplore this denigration of theory and the academy. And while real-world problems often have no single answer, mathematical problems do.

There must be a better compromise between the dry lecture and student-directed brainstorming than what is implied by this *Framework*.

As to the relative absence of Content in this *Framework,* the authors of this report are not saying that no mathematics is being taught in the Rhode Island schools, or that this *Framework* intends such a thing. But what is written in the "Process and Content Standards" is so vague that it is impossible to tell. In K-4, it is literally not clear at all if the student really knows how to add, subtract, multiply and divide, or just has an "appreciation" for those things. What part of these four operations shall children memorize? What algorithms are taught, and what schemes are left to their own construction? The *Framework* offers no answer, but the teachers across the state need one.

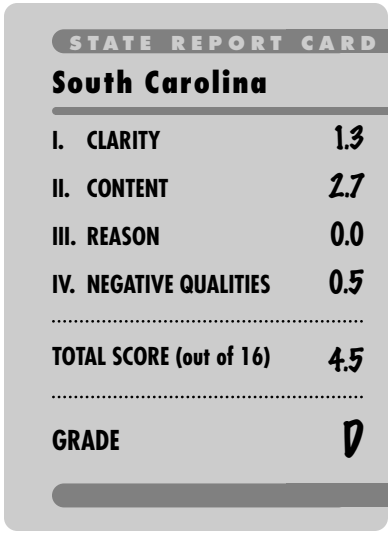
The Constructivist stance elaborated throughout the *Framework* would, if diligently followed, prevent a full curriculum from taking hold, since there is not time for the students to construct all the required knowledge for themselves. It might be that Rhode Island does not intend it so, but the vignettes contributed by teachers to illustrate ideal lessons argue that they, at least, are taking the idea seriously, so much so that one of them (at a junior high school) announced that, among other things, "Students also discovered Euler's formula, F+V-2=E." In truth, the formula is simple enough for children to understand, even if its proof is hazardous; but to say the children "discovered" it, or to have made them believe so, has its own dangers. True discovery is not taught by surreptitiously feeding answers. There must be a better compromise between the dry lecture and student-directed brainstorming than what is implied by this *Framework.*

STATE REPORT CARD

Rhode Island

I. CLARITY	0.0
II. CONTENT	1.0
III. REASON	0.0
IV. NEGATIVE QUALITIES	0.0
TOTAL SCORE (out of 16)	1.0
GRADE	F

South Carolina



The *Standards* is a revised version of the relevant portions of a *Framework,* published earlier and containing much besides content standards, some of it illuminating the intention behind the *Standards,* which is the principal subject of this evaluation. In the *Standards* are found fairly demanding curriculum indications, especially concerning numerical and algebraic concepts for the early grades (p. 22), and concerning numerical systems at the middle school level (p. 38). On page 66 ("Geometry," high school) occurs a more typical—i.e., ambiguous—demand: "Deduce properties of and relationships between figures from given assumptions by: using logical reasoning to draw conclusions about geometric figures from given assumptions." The suggested method (the only one mentioned there) uses the same words as the thing it is supposed to be a method for, and so does not add anything. But the real ambiguity is the question of the depth of investigation asked for. It could be anything from "math appreciation" to a full-scale course in Euclid. Items of this kind, which are numerous, render unclear what students are asked to accomplish, and might even render inaccurate our appraisal in this report of what "content" is being rated by our numerical scheme.

On the other hand, the instructions (p. 60) concerning the desired facility (grade 12 level) in manipulating algebraic expressions, and solving equations and inequalities, are well-stated and illustrated by examples, though the content appears a bit thin. Binomial theorem? Quadratic formula? Proof of either? We don't know. To add to the uncertainty, the phrase "using appropriate technology" appears repeatedly throughout the entire document, in every context. The teacher is thus deprived of necessary guidance as to the balance between what is to be internalized by the intellect and what is to be consigned to machinery.

The educational philosophy expressed in the *Framework,* though not a subject of numerical evaluation in this report, includes this curious observation, which might illuminate the vision of past practice in mathematics education that underlies the recommendations in the present *Standards:* "Mathematics should be a discipline that helps [students] to make sense of things, not a discipline that is arbitrary and devoid of meaning" (p. 11). We believe that the presumption that people, and in particular teachers, have in the past thought the latter is unwarranted.

Under False Doctrine of another sort is the instruction on page 10 of the *Standards* volume, concerning geometry and spatial sense for primary grades students: "Students at the primary level are not expected to know these words." It is not clear which words are in question, but the children are, in the sentence following, expected to do things with tessellations, symmetry, congruence, similarity, scale, perspective, angles, and networks; why should they be deprived of an adequate vocabulary? Grades 7 and 8, on the other hand, sometimes apparently call for too much, or too advanced, material: fractals, Fibonacci, the Golden Mean, and permutations (e.g., to calculate nPr, albeit with technology). At that level, fractals can only be "math appreciation," and nPr will never be assimilated.

South Dakota

The introduction tells of the purposes of standards, in particular, that they are not to be construed as a curriculum, but that they "make an effort to describe the 'big ideas' or core content which should be learned by all students. . . . Content standards provide guidelines for the development of curriculum by listing important areas of content which must be addressed."

Further, "These standards compel teachers not only to look at the mathematics taught but at what is learned, and at why it is learned." And, "Checklists of concepts and skills are no longer the primary component of the mathematics curriculum." The examples of standards and the more particular "benchmarks" listing what a student should know or do at particular grade levels, to be quoted below, will indicate that these purposes are not met by the present document.

There are six standards, (here) briefly labeled algebra, geometry, measurement, numbers, functions, and statistics. Each standard is printed at the top of a new page and followed by a "Rationale," a paragraph telling of its importance and place in the curriculum, and mentioning such things as societal needs, real-life experiences, connections across the curriculum, our daily lives, etc. Finally, the "benchmarks" offer what specifics the document contains; they are all prefaced by "Students will. . . ." The quotations given below will mention the grade level intended for each. We hope the vacuity of each of them as a guide for teachers or for curriculum development will speak for itself.

- Model number relationships using a variety of strategies (3-4);
- Analyze and describe situations that involve one or more variables (9-12);
- Compare and contrast spatial relationships using geometric figures (3-4);
- Use models, manipulatives, and computer graphics software to build a strong conceptual understanding of geometry and its connection to other mathematics strands, science and art (5-8);
- Use models, manipulatives, and computer graphics software to build a strong conceptual understanding of trigonometry and its connection to other mathematics strands, science and art (9-12);
- Measure quantities indirectly using techniques of algebra, geometry, or trigonometry (9-12);
- Utilize the real number system to demonstrate problem-solving strategies (5-8);
- Determine and describe the effects of number operations on real numbers with varying magnitudes (9-12).

Many of the words used in these statements, and all the others, are marked as being defined in the Glossary (page M 14). Some of the definitions are misleading, at the least. "Number Systems" is defined as "systems of numbers including, but not limited to, complex, discrete, and binary number systems." But "binary" is a way of writing real numbers, not a system of numbers; the given definition is analogous to saying that whole numbers, fractions, and Roman numerals are three kinds of number systems. It is downright destructive of mathematical understanding to teach children that a typographical artifact, such as binary notation, is a fundamental mathematical system, like the complex numbers.

One more quotation before we leave: A "benchmark" on page M 11, grade level 9-12, states that "students will demonstrate the concept of limit using mathematical models." We have not been able to determine the meaning of this instruction.

STATE REPORT CARD

South Dakota

I. CLARITY	0.0
II. CONTENT	0.0
III. REASON	0.0
IV. NEGATIVE QUALITIES	1.5
TOTAL SCORE (out of 16)	1.5
GRADE	F

Tennessee

The *Framework* for grades K-8 is a 1996 document while the *Curriculum Framework* for grades 9-12 is dated 1991. Tennessee states that the latter is now under revision. Since they are so different in every way, obviously written by people with vastly different purposes, we are giving Tennessee two separate sets of ratings. Averaging them does not honestly reflect our estimate of either.

The K-8 *Framework* begins with four "Process Standards": Problem Solving, Communication, Reasoning, and Connections, each incorporated into each of seven "Content Standards": "Number Sense and Number Theory"; "Estimation"; "Measurement and Computation"; "Patterns"; "Functions and Algebraic Thinking"; "Statistics and Probability"; and "Spacial Sense and Geometric Concepts." Then there is a set of five learning "expectations," and a Glossary for terms that appear in the document.

Some idea of the educational philosophy underlying the entire *Framework* may be had from the Glossary entries. The authors warn that these definitions are not complete, or designed for student use, but that "the definitions given are restricted to only the common mathematical meaning." It is further explained that "the terms and definitions included in this Glossary were produced by the *Mathematics Curriculum Framework Committee for Grades K-8.*" Here are a few of these definitions:

Algebraic thinking—thinking skills which are developed by working with problems which require students to describe, extend, analyze, and create a variety of oral, visual, and physical patterns (such as ones based on color, shape, number, sounds) from real life and other subjects such as literature and music.

Equation—two mathematical expressions joined by an equals sign.

Model—(verb) to show or illustrate a concept or problem by using physical objects with manipulations of these objects; to use simpler or more familiar objects and situations to explain a new concept, to solve a given problem, or to demonstrate understanding of a concept.

The first of these definitions ("Algebraic thinking") shows the marks of the deplorable current fashion emphasizing the primacy of "real-life" applications of mathematics to such a degree that mathematics is deliberately confused with the world it seeks to model. Except for "thinking skills," a nebulous phrase in itself, there is absolutely no reference here to anything remotely mathematical, symbolic, or deductive, let alone algebraic.

The definition of "equation" is amusing; it is a definition of what an equation looks like on the printed page, perhaps, but seems not to understand that an equation is a certain sort of statement, a sentence written in English. The sad state of public understanding of mathematics can be traced to the early acquisition of such "definitions," which can only lead to meaningless manipulation of symbols, if that.

The definition of "model" shows a misunderstanding of the role of mathematics in modeling phenomena; the given definition totally reverses the process, betraying the notion in the mind of the writer that the mathematics is the difficult thing, and the real world the simpler, by which the mathematics is to be elucidated. This may be true for very young children, whose understanding of arithmetic must of course be rooted in experience, but suggests that the authors of these Frameworks wish to convey an understanding of mathematics and its relation to the real world that stops at that point. Certainly, they are ignoring the current use of "model" in mathematics: as a verb it refers to the use of mathematics to picture real phenomena, while as a noun it refers to the mathematical structure that serves in place of the reality. This definition has it backwards.

A study of this Glossary does not cause one to anticipate much mathematical content in the Framework itself, for all that it lists an enormous number of rubrics by which to classify what happens in grades K-8. There are also some goals: "Learning to value mathematics"; "Becoming confident in one's own ability"; "Becoming a mathematical problem solver"; "Learning to communicate mathematically"; and "Learning to reason mathematically." Yet with all these rubrics, there turn out to be only 12 pages of non-repeated text material in this 46 page *Framework.* These convenient pages order the items by grade level, with the mathematical categories listed within each level. Here is the place, if any, to give substance to the philosophy and categories of the surrounding texts. We are willing to tolerate vagueness in a title or category description, provided the text ultimately makes plain what the generalization describes. In the K-8 *Framework,* however, most generalizations are not later made more specific.

A typical "learning expectation" from the K-8 document concerns "Patterns, Functions and Algebraic Thinking" for the Grade 6-8 level: "Describe, extend, analyze, and create a wide variety of patterns in numbers, shape, and data using a variety of appropriate materials, including manipulatives and technology."

The phrases, "a variety of . . ." and "a wide variety of . . .," as emphatic modifiers of perfectly understandable plurals, are frequent enough in many standards documents, but more frequent in this K-8 *Framework* than usual. They add nothing to the statement but words; they are inflation.

Coming to the level Grades 3-5, now, under "Number Sense and Number Theory," we find that students should

STATE REPORT CARD

Tennessee (K-8 only)

I. CLARITY	0.0
II. CONTENT	1.5
III. REASON	0.0
IV. NEGATIVE QUALITIES	1.0
TOTAL SCORE (out of 16)	2.5
GRADE	F

STATE REPORT CARD

Tennessee (9-12 only)

I. CLARITY	4.0
II. CONTENT	4.0
III. REASON	3.0
IV. NEGATIVE QUALITIES	3.0
TOTAL SCORE (out of 16)	14.0
GRADE	A

- Articulate and model the relationship between fractions and decimal notation for a rational number;
- Select a notation and justify its appropriateness for a particular situation;
- Demonstrate an understanding of decimals by extending whole number place value concepts;
- Use geometric models to illustrate properties of whole numbers and fractions and their operations.

Here the second expectation is opaque to the outsider, including the professional mathematician. The fourth expectation is indefinite enough to be the basis for either a good lesson or a poor one. The first and third expectations are appropriate but repetitious; one cannot do the first without the understanding demanded in the third.

The 9-12 *Framework* is a document of comparable length, though without all the rubrics and repetitions of the other; hence it contains more. It outlines one sequence, "Arithmetic 9" (remedial), "Applied Mathematics I" (also remedial), and "Applied Mathematics II" (partly remedial) as evidently designed for the non-college bound students; another, consisting of "Algebra I," "Algebra II," and "Unified Geometry," is the standard pre-college curriculum that has traditionally occupied grades 9, 10, and 11 in many parts of the country.

There are descriptions of yet other courses ("Pre-algebra," for example) which fit into alternate tracks, all of which are outlined in such a way that students showing unusual proficiency, or difficulty, have a key to an appropriate next step in a different track, much as one would find in a college catalogue. As a whole, the 9-12 document spends very little space on "overarching" category listings. Algebra is algebra, geometry geometry and trigonometry trigonometry. There is no talk of "patterns," no "spatial sense and geometric concepts," and no "mathematics as communication."

For example, under Content for "Algebra II" we have a section on the complex numbers, with the direction, "Simplify complex fractions involving complex numbers." This is something that can hardly be done by people who have scanted computation with ordinary real fractions in favor of decimal representation and machines. The instruction is clearly stated and substantial in content, unlike so much in the K-8 listings. Other typical 9-12 demands, from various courses and levels are

- Construct a line parallel to a given line;
- Transform expressions with rational exponents into simplest radical form and conversely;
- Solve simple exponential and logarithmic equations;
- Classify a system of equations in three variables as consistent, inconsistent, dependent, [or] independent;
- Use the Gaussian elimination method.

This last skill can be made to seem unnecessary and old-fashioned to a student with a suitable calculator and the mandate to use it on all occasions. But it is only as "foolish" as learning multiplication tables. The Gaussian elimination is like walking, where a calculator is more like an airplane. But one cannot fly an airplane next door.

Not everything old-fashioned is good, though, and certain truly pointless old topics appear here and there in Tennessee's 9-12 *Framework.* "Synthetic division" for example, is a lavender-scented relic of forgotten ballrooms and will surely not survive the revision now under way. Descartes' Rule of Signs is another fossil which is seldom accompanied by an understandable rationale in school instruction, and at best gives insights of little value. (And despite all the changes of the past 50 years, Tennessee is not the only state to retain these two topics.) Also, there are some important things missing in the present 9-12 curriculum because they have always been missing; precedent is not always a good guide.

Thus, in our Table of Ratings in Section VII we omit grading the K-8 *Framework* for "Content" at the "Secondary" level, and we omit grading the 9-12 *Framework* for "Content" at the two earlier levels, assigning "n" for "no grade" in those places. Then we average what is left, giving four totals for each document as if it were a state unto itself. At the end there is the grade of C, corresponding to an overall, though misleading, "average" for all of Tennessee.

Texas

This document is particularly lucid and easy to read, though our copy lacks page numbers. It takes the curriculum grade-by-grade for K-8, and by course titles thereafter, a logical choice, and one which avoids the fragmentation of the later curriculum so common in "integrated" high school mathematics courses. The specifications of what is demanded at each level and each subject heading are introduced with useful summaries telling the reader where the document is and where it is going; and then the specific items are indeed specific, as they should be. The content is better than average, with the important omission of almost everything having to do with mathematical reasoning. There are other omissions concerning content, sometimes requiring the reader to *infer* it—for example, that the quadratic formula is to be proved and not just used from memory is inferred from the mention of "completing the square" as a method of solving a quadratic equation. In pre-calculus there appear to be no trigonometric identities, and the binomial theorem gets only brief mention; in these cases we infer less rather than more.

While the word "reasoning" appears frequently, it most often refers to making connections between the real world and its mathematical models, rather than to the logical connection between mathematical statements. Thus, "uses pictures or models to demonstrate the Pythagorean theorem," and "use the Pythagorean Theorem to solve real-life problems," ("Geometry," 8th grade) are not followed up by a demand for its proof, or even its placement in an organized mathematical system—not even where axiomatic systems come under discussion in grades 9-12. There the course description leaves unclear what sorts of proofs, if any, will be produced or learned. The "multiplicity of approaches" to geometry here does not outline a coherent course of study. For example, "In a variety of ways, the student develops, applies and justifies triangle similarity relationships, such as right triangle ratios, trigonometric ratios, and Pythagorean triples" gives a confused message. Pythagorean triples are of algebraic interest and teach little about geometry, and ". . . develops, applies, and justifies triangle similarity relationships, such as . . . Pythagorean triples" doesn't quite make sense.

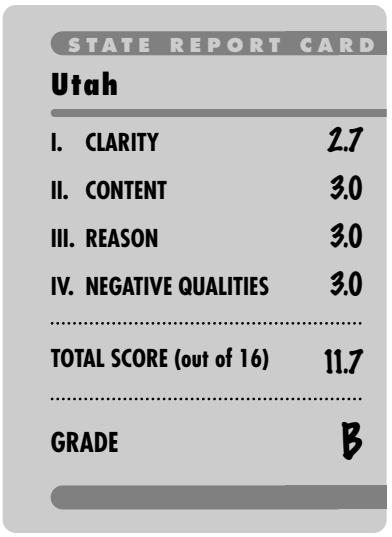
STATE REPORT CARD

Texas

I. CLARITY	3.1
II. CONTENT	3.0
III. REASON	1.0
IV. NEGATIVE QUALITIES	3.5
TOTAL SCORE (out of 16)	10.6
GRADE	B

The geometry course, little as its description implies as to content, goes so far as to suggest that non-Euclidean geometries are to be studied; this won't do, when even Euclidean geometry gets such short shrift. Texas is not the only state whose standards make such a recommendation (see Notes for Arkansas and Hawaii above).

Utah

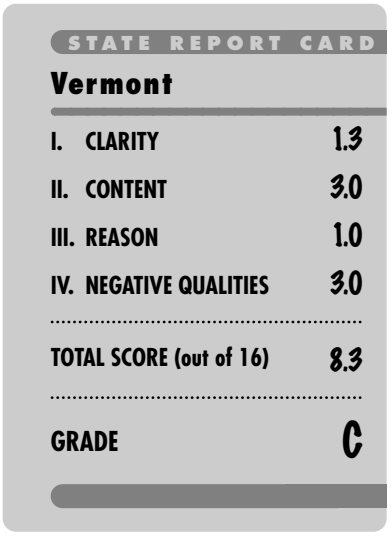


The organization of this document emphasizes the continuation of strands through several grade levels to such a degree that particular instructions are sometimes mistakenly placed. Thus Standard 5000, is intended for kindergarten, 5010 for grade 1, 5020 for grade 2, etc. For each grade level there are strands, e.g. Strand 12 having to do with fractions and decimals. Thus 5030-12 names an instruction for the 3rd grade level concerning fractions and decimals, and 5000-12 the same but for kindergarten. The following instruction might be appropriate for 5040-12, and is given there: "Develop concepts of fractions, mixed numbers, and decimals." But the same sentence appears also at 5000-12, 5010-12, 5020-12, and 5030-12, making it appear that decimal fractions are a feature of kindergarten instruction and then repeated each year for the next four years in identical terms. It is careless of the writers to appear to suggest this.

Each strand, at each grade level, begins with very general "objectives," which cannot be of much value—e.g., 5040-07 begins with "Develop meaning for the operations by modeling and discussing a rich variety of problem situations." (This is 4th grade level, for the strand concerned with what ordinary people call the arithmetic of whole numbers.) However, after the objectives come the "Skills and Strategies" section, offering particulars. In the case of 5040-07, one of them is "Demonstrate through the use of manipulatives that multiplication and division are inverse operations . . . (3X4=12; 12/4=3)," which could not be clearer or more definite. This is general throughout the document, and a good program is defined. Even Reason is well treated from time to time, as in 5350-14, "Recognize the applications of field properties in solving equations and inequalities," a recognition not often enough referred to in most states' standards.

On the negative side, the document is entirely too long and repetitious, the same instruction occurring in the same words again and again, so that the reader will be unable to decide the proper placement of skills, or unable to see the desired progression of skill as students progress through the grades. This is a species of Inflation, and another is the curious jargon that appears repeatedly, as if the processes of arithmetic were more mysterious than they are. In 5350-14 it is asked that students "solve problems with real numbers using Venn diagrams" when all that is meant (we believe) is that students should recognize the inclusion relations between, say, the set of rationals and the set of reals. The word "strategy" turns into a quite unacceptable technical term in 5010-07: "Recognize and employ the strategy that division by zero is undefined. (You do not divide by zero.)" The parenthetical command carries the message, after all, without any strategy. The following item in the same "skills and strategies" section says, "Recognize and employ the strategy that when zero is a factor, the product is zero." This is no more or less a fundamental fact of multiplication than that 3X4=12, and it certainly is not a "strategy." Inflation again. A careful pruning of about half the words, or maybe two-thirds of them, in these core standards would make a much better program description, for the scatter-shot listings do harbor very good curriculum choices.

Vermont



Only three pages are given to mathematics, yet the generalities, while often indefinite, portray an adequate curriculum up to the high school level. The high school descriptions, however, permit too many interpretations. The high school student is to "understand the basic structures of number systems," for example—possibly a large order and possibly not, depending on what is meant. On the other hand, arithmetic at K-4 and at 5-8 are reasonably outlined, and include negative numbers, all rational numbers, and the mention of nonrepeating decimals. The entire document is too brief to give much detail, yet some particular indicators outline a strong primary school program. Negative numbers, for example, are introduced in level K-4. The weakest thread is "Problem-Solving and Reasoning," especially at the high school level, where the text is impossibly general, and indeed inflated, e.g., "Work to extend specific results and generalize from them; and gather evidence for conjectures and formulate proofs for them; understand the difference between supporting examples and proof." That last clause is excellent; it is a pity that its points of reference are not hinted at. One does not make proofs; one makes proofs *about* things, and those things should be named in a sufficient standards document.

Virginia

This standards document is so well written, definite, and free of jargon and inflation, that one is disappointed at the modesty of its expectations. Despite repeated disclaimers that "technology shall not be regarded as a substitute for a student's . . . proficiency in basic computations," there is excessive emphasis on technology throughout, e.g., under Trigonometry": "Graphing utilities . . . provide a powerful tool for solving/verifying trigonometric equations and inequalities" (p. 22). If "solving/verifying" is related to "proving," this is not good advice. If only verification is wanted, it is probably easier to ask the teacher. On page 10, at grade 4, standard 4.8 prescribes calculators for products of two three-digit numbers, at grade 5 any divisor of more than two digits will call for calculator assistance (standard 5.5, p. 11), and at grade 6 and presumably thereafter, adult status having been reached, any divisor of more than one significant figure will demand calculators (standard 6.6, p. 13).

This standards document is so well written, definite, and free of jargon and inflation, that one is disappointed at the modesty of its expectations.

At the high school level, in "Algebra II," linear systems are only to be solved by matrix inversion using calculators, and the entire mathematical meaning of this part of algebra is thereby bypassed, including such practical applications as the recognition of linear dependency and the analysis of systems whose matrices are not square (p. 21). These are not particularly sophisticated matters, but they are fundamental. If linear algebra is to be taught at all, in even so simple a matter as discovering the intersection of three planes in space, the logic of the elimination of variables is of greater importance than the production of a list of numbers, which can be found by looking at the answer book, after all. Thus, Virginia is of divided mind in its advocacy of Reason, for on page 20 it also says, "The student will construct and judge the validity of a logical argument consisting of a set of premises and a conclusion . . ." and goes on in some detail. When, as in the case of the set of linear equations just mentioned, a golden opportunity arises for using a deductive argument, the state misses the opportunity.

An "Advanced Placement" calculus course is described, but the rest of the high school curriculum as outlined is not a sufficient preparation for this level of work. In particular, the lack of needed exercise in mathematical reasoning in most of the curriculum, as the example of linear algebra indicates, is also visible in the treatment of geometry.

STATE REPORT CARD

Virginia

I. CLARITY	4.0
II. CONTENT	2.3
III. REASON	2.0
IV. NEGATIVE QUALITIES	3.5
TOTAL SCORE (out of 16)	11.8
GRADE	B

One unusually good feature of these standards is the repeated instruction concerning speech and vocabulary, that the student should use technical words fluently and correctly, and explain his work. A good exercise, and a model for other good exercises, is found on page 13, item 6.3, "The student will explain orally and in writing the concepts of prime and composite numbers." Again, under "Patterns, Functions, and Algebra," 7th grade level, "The student will use the following algebraic terms appropriately in written and/or oral expression: equation, inequality, variable, expression, term, coefficient, domain, and range" (p. 16). However, in speaking mathematically here the authors should have followed their own advice, and used the mathematical "or," which means the same thing, we believe, as their "and/or."

Washington

Standard 1 ("The student understands and applies the concepts and procedures of mathematics") has five components, of which 1.2 ("Understand and apply concepts and procedures from measurement") occupies page 57. Now component 1.2 has three sub-components, one of which, named "Approximation and Precision" has two entries at each benchmark (i.e., grade level). The second entry in each benchmark is

At grade 4: estimate to predict and determine when measurements are reasonable, for example, estimating the length of the playground by pacing it off.

At grade 7: use estimation to obtain reasonable approximations, for example, estimating the length and width of the playground to estimate its area.

At grade 10: use estimation to obtain reasonable approximations, for example, estimating how much paint is needed to paint the walls of a classroom.

This sort of repetition is common throughout this nearly vacuous document, and is probably intended, by its slight variations, to suggest a progression of skills as the student grows older, but in few cases does it say anything instructive. In the case just quoted the three exercises are at least well-defined; more often they are so general as to defy interpretation. An example, at Standard 3 ("The student uses mathematical reasoning"), where component 3.3, headed "Draw Conclusions and Verify Results," prescribes at the 4th grade level, "reflect on and evaluate procedures and results in familiar situations"; at the 7th grade level, "reflect and evaluate on [sic] procedures and results in new problem situations"; and at the 10th grade level, "reflect on and evaluate procedures and results and make necessary revisions" (p. 63).

This document contains standards for other subjects besides mathematics, which takes up only 12 pages. It is marked "Washington State Commission on Student Learning, APPROVED—February 26, 1997," but the interior pages are marked "Work in progress." Since the present edition does contain *some* content, as for example in "Geometry" (p. 59), it is possible that the impulse behind the present adherence to vague generalities is not final state policy, and that future editions will contain even more that a teacher or textbook committee can actually use.

STATE REPORT CARD

Washington

I. CLARITY	0.0
II. CONTENT	1.0
III. REASON	0.0
IV. NEGATIVE QUALITIES	1.5
TOTAL SCORE (out of 16)	2.5
GRADE	F

West Virginia

This document is intended, among other things, as a guide to statewide assessment, and places in boldface, for that purpose, certain crucial items. The language is everywhere direct and usually quite definite, e.g., for 1st graders, "Given two whole numbers whose sum is 99 or less, estimate the difference, and find the difference using various methods of calculation (mental computation, concrete materials, and paper/pencil)" (p. 46). Such clarity is maintained throughout, though it sometimes makes the document sound like the table of contents, or sometimes exercises, of a series of textbooks. At the high school level, in "Algebra II," standard A2.9 asks the student to "perform basic matrix operations and solve a system of linear equations using the inverse matrix method. Graphing calculators will be used to perform the calculations." This is definite and clear, but not advisable, except as part of a more comprehensive treatment of systems of linear equations, as has been commented above (see the note for Virginia).

Furthermore, clarity and definiteness are not always present; one of the boldface instructions at the high school level is "solve problems using non-routine strategies" (p. 179). Even though this instruction occurs within the course description "Algebra II," it is not possible to tell what is meant; yet the state intends to examine students on this matter.

There is a thread called "Computer/Technology" that appears at every grade level, K-8, and is apparently designed to make sure children learn to use calculators or computers. It is unfortunate that these topics appear within the "mathematics" portion of the *Instructional Goals and Objectives,* where they might give the unwary teacher the impression, sometimes mistaken, that these sometimes sociological exercises are designed to augment mathematics lessons. "Identify work produced by using technology as intellectual property and thus protected [by] copyright laws" (grade 7, p. 121) is plainly not mathematics. This entire thread should be thought through again *as it is related to the mathematics curriculum.* A distinct course, perhaps called "Computer Science," teaching children how to use computers and how to behave in their presence, would be another matter, of course.

STATE REPORT CARD

West Virginia

I. CLARITY	3.3
II. CONTENT	3.7
III. REASON	2.0
IV. NEGATIVE QUALITIES	3.5
TOTAL SCORE (out of 16)	12.5
GRADE	B

Wisconsin

This 1997 "Standards" is a draft; the final version will serve as a syllabus for statewide tests; page xi states, "If content does not appear in the academic standards, it will not be part of a WSAS test." Some things that do not appear are the quadratic formula, the binomial coefficients, a geometric series, a Euclidean proof, a conic section, an asymptote. The curriculum, in other words, is weak; even the Pythagorean theorem comes in for only casual mentions, and the trigonometric functions appear only in the mention of "trigonometric ratios" in right triangles, as an incident in geometry to be mastered by the 12th grade level.

On page 27, "Geometry and its study of shapes and relationships is an effort to understand the nature and beauty of the world. While the need to understand our environment is still with us, the rapid advance of technology has created another need: to understand ideas communicated visually through electronic media. For these reasons, educated people in the 21st Century need a well-developed spatial order to visualize and model real-world problem situations."

Such a definition reduces geometry to a sort of empirical science in the service of graphic art, and shows nothing of the nature of the deductive structure which launched modern science. On page 29, the ideas of geometry are confused with the artifacts of "analytic geometry," as if *slope* and *intercepts* were geometric objects.

While the definition (called "Rationale") for geometry misses the point of mathematics as a model for structural relations, there is not in fact much Inflation or False Doctrine in this document. It is praiseworthy that, on page 22, it is warned that "the tools of technology are not a substitute for proficiency in basic computational skills." Yet the document is evasive on this point, as where it is advised that "selecting and applying algorithms for addition, subtraction, multiplication, and division" is given equal billing with "using a calculator" (p. 25). If one wants an answer to the question of whether children are taught "long multiplication," the answer will not be found here.

The document is similarly unclear on many other points, to the degree that one cannot really determine the suggested content at most levels. In particular, while logic is mentioned, there is no evidence that logical argument is ever associated with substantive mathematical information; all subject matter mentioned is presented without regard to the connections of one mathematical idea with another.

STATE REPORT CARD

Wisconsin

I. CLARITY	1.7
II. CONTENT	2.7
III. REASON	0.0
IV. NEGATIVE QUALITIES	3.0
TOTAL SCORE (out of 16)	7.4
GRADE	C

Japan

Japan has a delightful consistency throughout its document. In the primary grades, each grade starts with a list of objectives *for that grade*, followed by "Content" ("Numerical Calculations," "Quantities and Measurements," and "Geometrical Figures"), followed by a few paragraphs entitled "Remarks concerning Content." In grade 3, the students "learn how numbers are set on the abacus ("soroban"), and to use it in simple addition and subtraction"(p. 9). In grades 5 and 6, "Quantitative Relations" is added to the listings under "Content," followed by the usual "Remarks concerning Content." Here there is no misguided notion of having 9 or 10 "strands" running through all of the grades (some American standards documents consider that computing areas in the 1st grade should form part of a strand called "calculus.") In the middle school (grades 7-9) each grade has exactly the same format: "Objectives," "Content," "Remarks." All grade levels are refreshing to read; they are models of "clear, definite, testable," and no grade runs more than a few pages. In grades 10-12, the format is repeated, but by subject matter rather than by grade level. There is also an "honors" track for those who are academically inclined, as well as a *third* track, very rigorous, for those whose interests lie in science and engineering. At the end, there are very definite guides about the sequence of all the courses, taken separately or in parallel.

All this is achieved in a mere 47 pages, less than a *tenth* of the longest American framework, though "framework" is probably a good description of the Japanese document as well, for it does contain pedagogical hints of importance. In particular, consider this quotation from the "Second Grade Content":

> To enable children to develop their abilities to use addition and subtraction through getting deeper understanding of them.

which is followed by, *inter alia*

> To understand that addition and subtraction of 2- and 3- digits numbers are accomplished by using the basic facts of these operations for 1-digit numbers and to know and use them in column form.

It is typical of the Japanese standards to incorporate, as this example does, the essence of the reasoning into the implied lesson plan associated with the content to be conveyed. That is, "adding in columns" is not merely a practical necessity made obsolete by the electronic calculator, as many educators now believe, but is (when properly taught, of course) an elucidation of the nature of the decimal system, and an illustration of how mathematical reasoning proceeds from minimal information to what appears to be enormously more. In this case, the 9X9 addition table memorized by the end of the 2nd grade enables a child, provided with a bit of understanding of the nature of the system, to construct a table potentially infinite in scope. The writer of this item shows this understanding clearly, and many other examples can be quoted to show the same quality.

Calculators are not used before the 5th grade, to give children the chance to understand the decimal system first.

Items of content are clearly paced from grade to grade, with lists of vocabulary to be acquired by the end of the year given as further indication of what the year is supposed to accomplish. By the 3rd grade children are to understand decimal fractions, graphs, angles, "radius," the sphere. By the end of grade 4, they are to know

STATE REPORT CARD

Japan

I. CLARITY	4.0
II. CONTENT	4.0
III. REASON	3.0
IV. NEGATIVE QUALITIES	4.0
TOTAL SCORE (out of 16)	15.0
GRADE	A

rules for rounding off approximations, adding fractions with common denominator, multiplying and dividing two-digit numbers, and by the end of the 5th, congruence, the making of a statistical graph, least common divisors and multiples, and "to know that the result of division of whole numbers can always be represented as a single number using fractions." This last quotation also carries a lesson in concept of "number" which most American curricula either avoid or take for granted.

Calculators are not used before the 5th grade, to give children the chance to understand the decimal system first. One of the stated "objectives" at the grade 9 level includes, "[to] acquire the way of mathematically representing and coping with, and to enhance their abilities of mathematically considering things, as well as to help them appreciate the mathematical way of viewing and thinking." This much, taken alone, is vague (though poetic), but is followed by specifics enough: factoring, the quadratic equation, functional relations, the Pythagorean theorem, and so on, all presented in such a way as to follow "the mathematical way of viewing and thinking," much more than in such a way as to convince students of the "real-life" applications. Real life comes in its own place, a few years later.

APPENDIX: LIST OF DOCUMENTS REVIEWED

Alabama
Alabama Course of Study, MATHEMATICS, Mathematical Power K-12
(Alabama State Department of Education Bulletin 1997, No.4)
50 North Ripley St., P.O. Box 302102
Montgomery, AL 36130-2101

Alaska
(1) *Math/Science Framework* (1996)
(2) *Mathematics Performance Standards* (1997)
Department of Education
801 W. Tenth Street
Juneau, AK 99801-1894

Arizona
Mathematics Performance Objectives (March 11, 1997; adopted August 26, 1996)
Arizona Department of Education
1535 West Jefferson
Phoenix, AZ 85007

Arkansas
Mathematics Framework
General Education Division
Arkansas Department of Education
Four State Capitol Mall, Room 304A
Little Rock, AR 77201-1071
(From
http://arkedu.k12.ar.us/wwwade/sections/curframe/frame.htm;
July 22, 1997)

California
The California MATHEMATICS Academic Content Standards (Prepublication Edition, February 2, 1998) for Grades K-12
California State Board of Education
http://www.cde.ca.gov/board/board.html

Colorado
Model Content Standards (June 8, 1995)
Colorado Department of Education
Denver, CO, 80203

Connecticut
"Mathematics Curriculum Framework," Second Draft, August 7, 1997
Connecticut Department of Education
165 Capitol Avenue
Room 305, State Office Building
Hartford, CT 06106-1630

Delaware
Mathematics Curriculum Framework
Vol. 1 Content Standards (1995, revised March 1, 1996)
Delaware Dept. of Education
Dover, DE 19903-1402
(also http://www.dpi.state.de.us/dpi/standards/math)

District of Columbia
Mathematics-Science-Technology Curriculum Framework, Grades K-12, Revised Edition (undated, but post-1989)
District of Columbia Public Schools
415 12th Street, N.W., Suite 1209
Washington, D.C. 20004-1994

Florida
(1) *Sunshine State Standards and Instructional Practices, Mathematics* (May 29, 1996)
(2) *Florida Course Descriptions*, Grades 6-12, pp151-263 (1997)
Florida Department of Education
Capitol Building, Room PL 08
Tallahassee, FL 32301

Georgia
"Mathematics Quality Core Curriculum," Draft Revision, Edition 2 ("provided by the Georgia School Improvement Panel, February, 1997")
CD ROM available from the Georgia Department of Education
Telephone (404) 657-7411

Hawaii
(1) *State Commission on Performance Standards* (Final Report, June 1994)
(2) *Essential Content* (December, 1992)
Department of Education
OASIS/Systems Group
641 18th Avenue, Room V201
Honolulu, HI 96816-4444
or
Hawaii Department of Education
1390 Miller Street, #307
Honolulu, HI 96813

Idaho
K-12 Mathematics Content Guide and Framework (1994)
(in three parts: K-4, 5-6, and 7-12)
http://www.state.id.us

Illinois
Illinois Learning Standards (July 25, 1997)
Illinois State Board of Education
100 North First Street
Springfield, IL 62777

Indiana
Mathematics Proficiency Guide (Spring 1997)
Indiana Department of Education
State House, Room 229
Indianapolis, IN 46204-2798
(also http://doe.state.in.us)

Iowa
(Iowa apparently does not intend to publish a *Standards* or *Framework* of the sort that is under review in this report.)

Japan
Mathematics Program in Japan (Kindergarten to Upper Secondary School)
Japan Society of Mathematical Education (JSME), January, 1990
(Excerpt from the National Courses of Study, Revised by the Ministry of Education)
Published by: Japan Society of Mathematical Education
Private Postbox No.18, Koishikawa Post Office
Tokyo, Japan

Kansas
Mathematics Curriculum Standards (Revised July 1993, Reprinted October 1996)
Kansas State Department of Education
120 SE 10th Avenue
Topeka, KS 66612
(also http://www.ksbe.state.ks.us)

Kentucky
Core Content for Mathematics Assessment
http://www.kde.state.ky.us/caa/CCA/CCM1.html
Kentucky Department of Education
500 Mero Street
Frankfort, KY 40601

Louisiana
Content Standards Foundation Skills (May 22, 1997)
http://www.doe.state.la.us/os2httpd/public/contents.mframe.htm

Maine
Curriculum Framework for Mathematics and Science
(undated, but post-1995)
Maine Department of Education
23 State House Station
Augusta, ME 04333-0023

Maryland
(1) *Mathematics—A Maryland Curriculum Framework* (1985)
(2) *High School Core Learning Goals, Mathematics* (September 1996)
Maryland State Department of Education
200 West Baltimore Street
Baltimore, MD 21701-7595

Massachusetts
Mathematics Curriculum Framework (December 1995)
Commonwealth of Massachusetts
Department of Education
350 Main Street
Malden, MA 02148-5023

Michigan
Model Content Standards for Curriculum, including *Academic Core Curriculum Content Standards* (July 25, 1996)
http://cdp.mde.state.mi.us/ContentStandards/Mathematics/

Minnesota
"K-12 Mathematics Framework" (Draft chapters, 1997)
"Please do not quote, copy, or cite." Our copy was mailed to us from:
"Sci-Math, MN"
638 Capitol Square
550 Cedar Street
St. Paul, MN 55101

Mississippi
Mathematics Curriculum Structure (1995)
Mississippi Department of Education
550 High Street, Room 501
Jackson, MS 39201

Missouri
Missouri's Framework for Curriculum Development in Mathematics, K-12 (1996)
Missouri Department of Elementary and Secondary Education
205 Jefferson Street
Jefferson City, MO 65102

Montana
Framework for Improving Mathematics and Science Education (1996)
Montana Office of Public Instruction
PO Box 202501
Helena, MT 59620-2501

Nebraska
Mathematics and Science Frameworks for Nebraska Schools
(March 6, 1994)
Nebraska Department of Education
301 Centennial Mall South
P.O. Box 94987
Lincoln, NE 68509

Nevada
"We are in the process of developing new math standards during the 1997-1998 school year."

New Hampshire
K-12 Mathematics Curriculum Framework (February 1995)
New Hampshire Department of Education
101 Pleasant Street
Concord, NH 03301

New Jersey
(1) *Core Curriculum Content Standards for Mathematics* (1995, revised 1996)
http://www.state.nj.us/njded/education.htm
(2) *New Jersey Mathematics Curriculum Framework* (1996)
New Jersey Mathematics Coalition
http://dimacs.rutgers.edu/nj_math_coalition/framework

New Mexico
Content Standards with Benchmarks for Kindergarten Through 12th Grade (Fall 1996)
New Mexico Department of Education
300 Don Gaspar
Santa Fe, NM 87501-2786

New York
Learning Standards for Mathematics, Science and Technology (March 1996)
New York Education Department
Education Building
111 Washington Avenue
Albany NY 12234

North Carolina
Standard Course of Study and Grade Level Competencies, Mathematics K-12 (1992, 1993)
North Carolina Department of Public Instruction
301 N. Wilmington St.
Raleigh, NC 27601-2825

North Dakota
Mathematics Curriculum Framework Standards and Benchmarks (Revised 1996-1996; Draft in progress March 24, 1997)
North Dakota Department of Public Instruction
State Capitol Building, 11th Floor
Bismarck, ND 58505-0440

Ohio
Model Competency-Based Mathematics Program (November 1990)
Ohio Department of Education
65 South Front Street, Room 810
Columbus, OH 43215-4183

Oklahoma
Priority Academic Student Skills—Mathematics (March 1997)
Oklahoma State Department of Education
2500 N. Lincoln Boulevard
Oklahoma City, OK 73105-4599

Oregon
(1) *Standards* (January 1997)
(Mathematics pages are 9-12 and 29,30)
(2) *Oregon Statewide Mathematics Assessment, Test Specifications Grade 3, Grade 5, Grade 8, Grade 10* (1997)
(3) Sample Tests for (2)
(4) *Mathematics Teacher Support Package* (October, 1996)
Oregon Department of Education
255 Capitol St. NE
Salem, OR 97310-0203

Pennsylvania
"Proposed Academic Standards for Mathematics 'for the Governor's Advisory Commission on Academic Standards'"
Undated but clearly 1997, taken from
http://www.cas.psu.EDU/PDE.HTML on 8/14/97
Pennsylvania Department of Education
333 Market Street
Harrisburg, PA 17126-0333

Rhode Island
Mathematics Framework K-12 (October 1995)
Department of Education
225 Westminster Street
Providence, RI 02903

South Carolina
(1) *Mathematics Framework* (November 1993)
(2) *Mathematics and Academic Achievement Standards* (November 1995)
Curriculum Framework Office
1429 Senate Street
Columbia, SC 29201

South Dakota
Mathematics Content Standards (Approved June 17, 1996)
Division of Education Services and Resources
700 Governors Drive
Pierre, SD 57501-2291
(Note: A letter of August 18, 1997 states that "South Dakota is in the process of rewriting the Content Standards.")

Tennessee
(1) *Mathematics Framework/ Grades Kindergarten Through Grade Eight* (October 11, 1996)
(2) *Mathematics Curriculum Framework*, Grades 9-12 (November 15, 1991)
Tennessee Department of Education
Andrew Johnson Tower
Nashville, TN 37243-0375

Texas
Texas Essential Knowledge and Skills for Mathematics ("Chapter 111," to be implemented by September 1, 1998, with Chapter C, 9-12, "effective September 1, 1996")
Texas Education Agency
1701 North Congress Avenue
Austin, TX 78701-1494
http://www.tea.state.tx.us/teks

Utah
Core Curriculum/ Mathematics Units (September 19, 1996)
http://www.uen.org/cgi-bin/websql/lessons/query_lp.hts?corearea=2&area=1

Vermont
Vermont's Framework of Standards and Learning Opportunities—Science, Mathematics and Technology Standards (1996)
Vermont State Board of Education
120 State Street
Montpelier, VT 05620-2501
http://www.state.vt.us/educ/stand/smtstand.htm

Virginia
Standards of Learning for Virginia Public Schools (June 1995)
Commonwealth of Virginia Department of Education
101 N. 14th Street
Richmond, VA 23219

Washington
Essential Academic Learning Requirements/Mathematics (February 26, 1997)
Commission on Student Learning
Room 222, Old Capitol Building
Olympia, WA 98504-7220

West Virginia

Instructional Goals and Objectives for West Virginia Schools
(September, 1996)
West Virginia Board of Education
1900 Kanawaha Blvd. E.
Charleston, WV 25305-0330
http://access.k12.wv.us/~dshafer/pmat9-12.htm

Wisconsin

"Model Academic Standards for Mathematics" (Draft, 1997)
Wisconsin Department of Public Instruction
P.O. Box 7841
Madison, WI 53707

Wyoming

Standards are in progress and were not available for review at publication time.

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http://www.edexcellence.net

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